1. Derive weak form for

\[ \frac{d}{dx}(EA \frac{du}{dx}) = 0 \]  \text{ strong form} 

\[ \int_0^L N\delta(x)(EA \frac{du}{dx}) dx = 0 \]  \text{(1)}

\( E, A \) is constant along \( x \).

\[ \frac{d}{dx}(N \frac{du}{dx}) = \frac{dN}{dx} \frac{du}{dx} + N \frac{d^2u}{dx^2} \]

\[- N \frac{d^2u}{dx^2} = \frac{dN}{dx} \frac{du}{dx} - \frac{d}{dx}(N \frac{du}{dx}) \]

Substituting the above relation into \( \text{(1)} \)

\[- \int_0^L EA \frac{dN}{dx} \frac{du}{dx} dx + \int_0^L EA x \frac{dN}{dx} \frac{du}{dx} dx = 0 \]

\[- \int_0^L EA \frac{dN}{dx} \frac{du}{dx} dx = N(0) EA \frac{du}{dx}|_{x=0} - N(L) EA \frac{du}{dx}|_{x=L} \]

\[- \int_0^L EA \frac{dN}{dx} \frac{du}{dx} dx = N(0) F(0) - N(L) F(L) \]

where \( F(x) = AE \frac{du}{dx} \)
i) Displacement continuity at $L$ requires

$u^{(w)}(x=L) = u^{(w)}(x=0)$

$u^{(1)} = \frac{\Delta u}{\Delta x}$ at $x=L$

$u^{(o)} = \frac{\Delta u}{\Delta x}$ at $x=0$

ii) $\varepsilon_x = \frac{du}{dx}$

$\varepsilon_x^{(1)} = \frac{d\varepsilon_x^{(o)}}{dx} = -\frac{du}{L} + \frac{\Delta u}{L} = \frac{\Delta u - \Delta u}{L}$

$\varepsilon_x^{(o)} = \frac{d\varepsilon_x^{(o)}}{dx} = -\frac{du}{L} + \frac{\Delta u}{L} = \frac{\Delta u - \Delta u}{L}$
Element
1. $\theta = 120^\circ$
2. $\theta = 90^\circ$

\[ K^{(1)} = \frac{AE}{L} \begin{bmatrix} d_{xx} & d_{xy} & d_{xx} & d_{xy} \\ d_{xy} & d_{yy} & d_{xy} & d_{yy} \\ d_{xx} & d_{xy} & d_{xx} & d_{xy} \\ d_{xy} & d_{yy} & d_{xy} & d_{yy} \end{bmatrix} \]

\[ K^{(2)} = \frac{AE}{L} \begin{bmatrix} 1.25 & 0.43 & -0.25 & 0.43 \\ 0.43 & 0.75 & 0.43 & -0.75 \\ -0.25 & 0.43 & 1.25 & -0.43 \\ 0.43 & -0.75 & 0.43 & 0.75 \end{bmatrix} \]

\[ K = K^{(1)} + K^{(2)} = \frac{AE}{L} \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \end{bmatrix} \]

\[ K = K^{(1)} + K^{(2)} = \frac{AE}{L} \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{bmatrix} \]

+ see next page
\[ K = \frac{AE}{L} \begin{bmatrix} 0.25 & -0.43 & 0 & 0 & -0.25 & 0.43 \\ -0.43 & 1.75 & 0 & -1 & 0.43 & -0.75 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 1 & 0 & 0 \\ -0.25 & 0.43 & 0 & 0 & 0.25 & -0.75 \\ 0.43 & -0.75 & 0 & 0 & -0.43 & 0.75 \end{bmatrix} \]
\( \vec{\delta} = -f \)

\( \vec{d}_0 = 0 \)

\( \vec{d}_1 = 0 \)

\( \vec{d}_2 = 0 \)

\( \vec{d}_3 = 0 \)

\( \vec{m}_1 = 0 \)

\( \vec{m}_2 = 0 \)

\[
\begin{bmatrix}
    \begin{bmatrix}
        \hat{\delta}_{y}
    \end{bmatrix} \\
    \begin{bmatrix}
        \hat{m}_1
    \end{bmatrix}
\end{bmatrix} = \frac{EI}{L^3} \begin{bmatrix}
    12 & -12 & 6L \\
    -12 & 24 & -6L \\
    6L & -6L & 0
\end{bmatrix} \begin{bmatrix}
    \begin{bmatrix}
        \hat{d}_2
    \end{bmatrix} \\
    \begin{bmatrix}
        \hat{d}_3
    \end{bmatrix}
\end{bmatrix}
\]

\[
\begin{bmatrix}
    \begin{bmatrix}
        \hat{\delta}_{y}
    \end{bmatrix} \\
    \begin{bmatrix}
        \hat{m}_1
    \end{bmatrix}
\end{bmatrix} = \frac{EI}{L^3} \begin{bmatrix}
    12 & -12 & 6L \\
    -12 & 24 & -6L \\
    6L & -6L & 0
\end{bmatrix} \begin{bmatrix}
    \begin{bmatrix}
        \hat{d}_2
    \end{bmatrix} \\
    \begin{bmatrix}
        \hat{d}_3
    \end{bmatrix}
\end{bmatrix}
\]

\[
\begin{bmatrix}
    \begin{bmatrix}
        \hat{\delta}_{y}
    \end{bmatrix} \\
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    \end{bmatrix}
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    12 & -12 & 6L \\
    -12 & 24 & -6L \\
    6L & -6L & 0
\end{bmatrix} \begin{bmatrix}
    \begin{bmatrix}
        \hat{d}_2
    \end{bmatrix} \\
    \begin{bmatrix}
        \hat{d}_3
    \end{bmatrix}
\end{bmatrix}
\]
\[ \begin{align*}
\{f\} & = \frac{EI}{l^3} \begin{bmatrix} -12 & -6L & 12 & -6L \\ -6L & 2L^2 & -6L & 4L^2 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \\
\{f\} & = \frac{EI}{l^3} \begin{bmatrix} 12 & -6L \\ -6L & 9L^2 \end{bmatrix} \begin{bmatrix} x \\ \phi_2 \end{bmatrix} \\
\{x\} & = \frac{l^3}{EI} \begin{bmatrix} 1/3 & 1/2L \\ 1/2L & 1/2L \end{bmatrix} \begin{bmatrix} f \\ 0 \end{bmatrix} \\
\phi_2 & = -\frac{1}{EI} \frac{1}{y} \\
\phi_2 & = \frac{1}{EI} \frac{1}{y} \\
\phi_2 & = \frac{L^3}{EI} \frac{-f}{2L} = -\frac{L^2f}{2EI} \\
\{y\} & = \frac{EI}{L^2} \begin{bmatrix} -12 & -6L \\ -6L & 2L^2 \end{bmatrix} \begin{bmatrix} x \\ \phi_1 \end{bmatrix} \\
\{y\} & = \frac{EI}{L^2} \begin{bmatrix} 12 & -6L \\ -6L & 9L^2 \end{bmatrix} \begin{bmatrix} x \\ \phi_2 \end{bmatrix} \\
\phi_1 & = \frac{9}{2} f + \frac{3}{2} f = 6f \\
\phi_1 & = \frac{9}{2} f + \frac{3}{2} f = 6f \\
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\phi_1 & = \frac{9}{2} f + \frac{3}{2} f = 6f \\
\phi_1 & = \frac{9}{2} f + \frac{3}{2} f = 6f \\
\hat{M}_1 & = \left( \frac{3}{5} - \frac{1}{2} \right) f \cdot l = \frac{1}{10} f \cdot l
\end{align*} \]