2. POWER TRANSFORMERS

2.1. Introduction

This chapter is intended to cover a major component of the power system, which is the power transformer. Transformers make large power systems possible. In order to transmit hundreds of megawatts of power efficiently over long distances, very high line voltages are necessary to decrease the line losses. Power at low voltages is also necessary to be used at a safe level in home appliances and most industrial equipment.

Power transformers can be classified as:

- Step-up transformers to be connected between the generator and the transmission line. They permit a practical design voltage for generators, and at the same time an efficient transmission line voltage.
- Step-down transformers connected between the transmission line and various electrical loads. They permit the transmitted power to be used at a safe utilization voltage.

2.2. Types of Power Transformers

The elements of a power transmission and distribution system are shown in Figure 2.1. The output terminals of generators are usually connected directly to a generator step-up unit (GSU) of equal rating. The GSU steps the voltage of the generator up to the desired transmission voltage. Figure 2.2 is a typical transformer. At the receiving ends of the transmission system are substations, at each of which there are one or more power transformers. They reduce the voltage to the sub-transmission levels. The sub-transmission circuits fan out from the substation to distribution substations located at load centers. At the load centers, small power transformers further reduce the voltage to distribution levels. Distribution circuits go to industrial loads or residential districts where the voltage is reduced to the final utilization voltage. The local transformers performing the final voltage reduction are called distribution transformers. A typical single-phase distribution transformer is shown in Figure 2.3

Three-phase power may be transformed by using either two or three single-phase transformers, or by a single, three-phase transformer. When a set of single-phase transformers is employed to transform a three-phase circuit, it is called a three-phase transformer bank.

Some other types of transformers are used in measuring voltage, current, and power flow in the power system. The majorities are potential transformers and current transformers. Potential transformers (PT) are single-phase transformers of special design, which step down the voltage to be measured to a safe value. Current transformers (CT) step down the currents and have insulation adequate to isolate metering equipment and personnel from the line voltage. One terminal of the secondaries of both potential and current transformers is usually grounded for safety.

2.3. Elements of Transformers

The transformer consists of two or more insulated windings wrapped around an iron core. By definition, the primary winding is the input winding, and the secondary winding is the output winding.

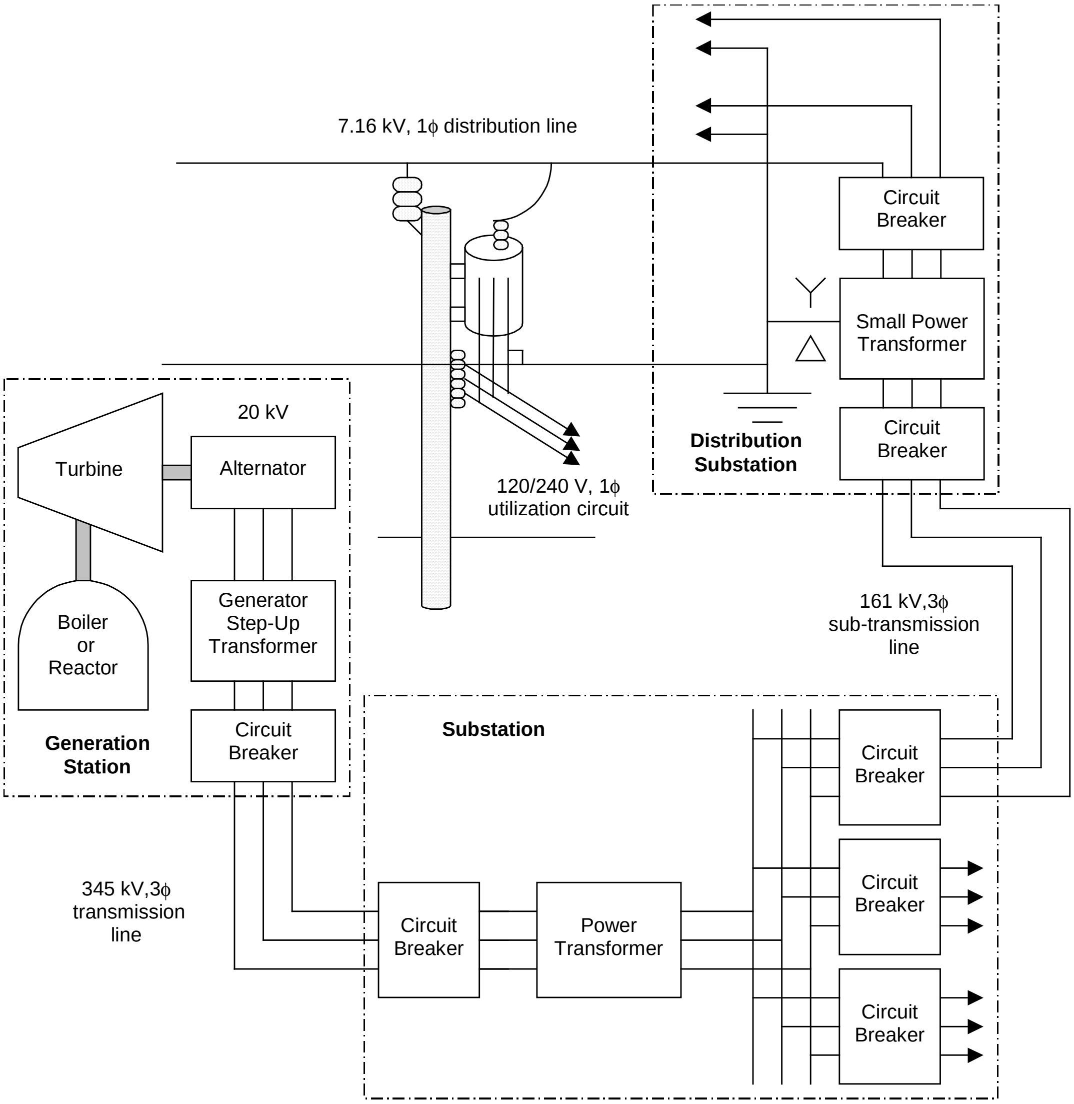


Figure 2.1. Elements of a Power Transmission and Distribution System.

Figure 2.2. A 500 MVA Power Transformer. (Courtesy of ABB)

Figure 2.3. A Pole-Mount 15 kVA Distribution Transformer. (Courtesy of ABB)

The core is formed of a stack of steel laminations. The steel has a high magnetic permeability and provides a high-performance path for the flux, which is mutual to the primary and secondary windings. The core is built up of thin laminations, which are electrically insulated from each other. Two types of core construction are used as shown in Figure 2.4. The first is known as the *core type* and the second is denoted the *shell-type*.

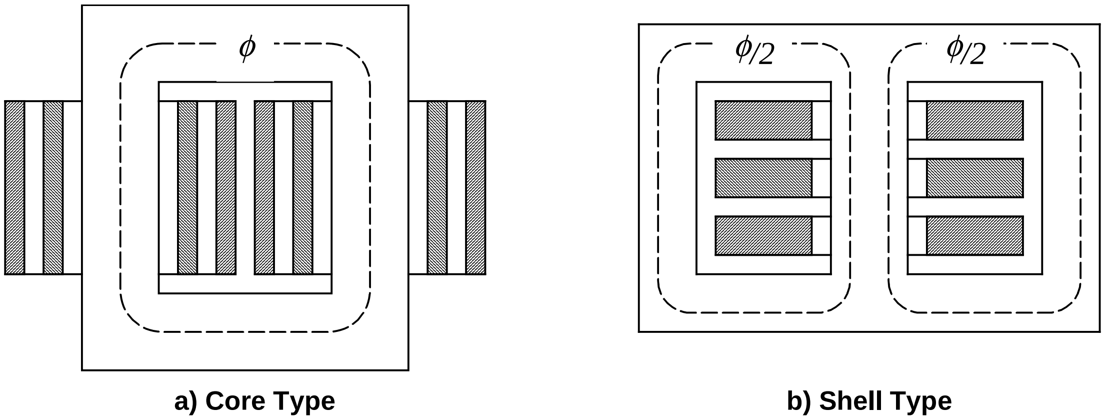


Figure 2.4. a) Core-Type, b) Shell-Type.

The three active elements of a transformer are the primary winding, secondary winding, and the core. There are three types of insulation-cooling methods. Dry-type transformers operate in air. Cast-Coil transformers are encapsulated in an epoxy resin. Most power transformers are immersed in a tank of oil. The oil is a better insulator than air. The ends of the windings are brought to a terminal block from which leads are brought to the outside of the tank through insulating bushings, mounted in holes in the sides or top of the tank. The high-voltage bushings are seen clearly in Figure 2.3.

When the primary is connected to an ac voltage source, an alternating flux is set up in the core. This flux induces voltages in all windings. The voltage induced in each winding, according to Faraday's Law, is proportional to the number of turns in that winding. The voltage induced in the primary is nearly equal to the applied voltage, and the voltage at the secondary-winding terminals also differs by only a few percent from the voltage induced into that winding. Thus the primary-to-secondary voltage ratio is essentially equal to the ratio of the number of turns in the two windings. The turns-ratio is given the symbol a.

$$a \equiv \frac{N_1}{N_2} \cong \frac{V_1}{V_2} \tag{2.1}$$

where N_1 is the number of primary coil turns, N_2 is the number of secondary coil turns, and V_1, V_2 are the voltages at the winding terminals. By selecting the proper turns-ratio, the transformer designer can determine the ratio of input to output voltages to meet the requirements of the power system.

2.4. General Theory of Transformer Operation

Transformers operate on the basis of Faraday's Law:

$$e = \pm\frac{d\lambda}{dt} = \pm N\frac{d\phi}{dt} \tag{2.2}$$

where e is the instantaneous voltage induced by a magnetic field, λ is the number of flux linkages between the field and the electric circuit in which the voltage is being induced, and ϕ is the effective flux. The sign depends on Lenz's Law and the polarity of the circuit terminals.

If the winding resistance is neglected, then

$$v_1 \cong e_1 = N_1\left(\frac{d\phi}{dt}\right), \quad v_2 \cong e_2 = N_2\left(\frac{d\phi}{dt}\right) \tag{2.3}$$

Taking the voltage ratio in Equation 2.3, we see that

$$\frac{N_1}{N_2} = \frac{e_1}{e_2} \tag{2.4}$$

Neglecting losses means that the instantaneous power is the same on both sides of the transformer, as

$$e_1 i_1 = e_2 i_2 \tag{2.5}$$

Combining Equations 2.1, 2.4 and 2.5, we get

$$a = \frac{N_1}{N_2} = \frac{v_1}{v_2} = \frac{i_2}{i_1} \tag{2.6}$$

As mentioned before, the transformer action requires the existence of the flux that links the two windings. This will be obtained more effectively if an iron core is used; because, an iron core confines the flux to a definite path linking both windings. A magnetic material such as iron undergoes a loss of energy due to the application of alternating voltage to its magnetization hysteresis loop (B-H curve) as shown in Figure 2.5.

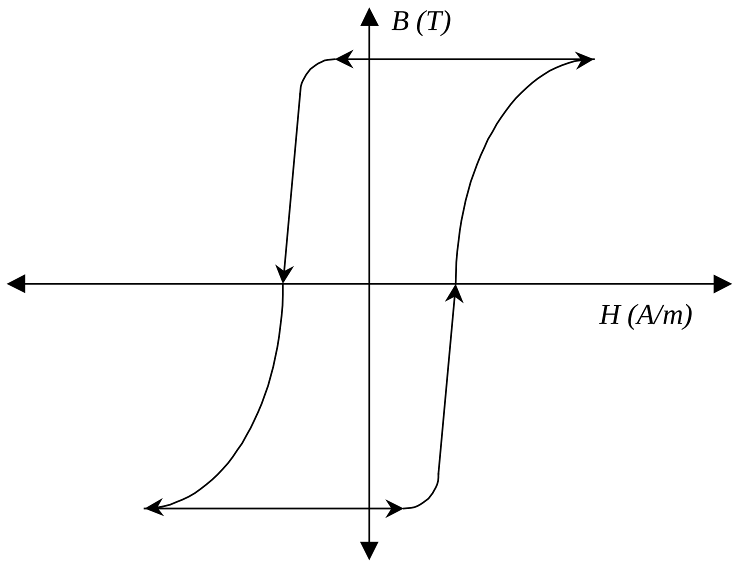


Figure 2.5. A Magnetic Hysteresis Loop or B-H Curve of Core Steel.

The losses are composed of two parts. The first is called the eddy-current loss, and the second is the hysteresis loss. Eddy current loss is basically loss due to the induced current in the magnetic material. To reduce these losses, the magnetic circuit is usually made of a stack of thin laminations. Hysteresis loss is caused by the energy used in orienting the magnetic domains of the material along the field. The loss depends on the material used.

Due to the non-linearity of the B-H curve of the magnetic material, the primary current will not be a sinusoid but rather a certain distorted version, which is still periodic. For analysis purposes, we approximate the current with a sinusoidal function of the fundamental frequency. This approximated primary current is made of two components. The first is in-phase with the voltage and is attributed to the power taken by eddy-current and hysteresis losses and is called the core-loss component, I_c, of the exciting current, I_ϕ or I_o. The second component lags the voltage by 90° and is called the magnetizing current, I_m.

2.4.1. Principle of Operation

In Figure 2.6 the basic components of the transformer are the core, primary winding, and secondary winding. If the flux ϕ_M is the mutual (or core) flux linking N_1 and N_2, then according to Faraday's Law of electromagnetic induction, electromagnetic forces (emf's) are induced in N_1 and N_2 due to a time rate of change of ϕ_M such that:

$$e_1 = N_1 \frac{d\phi_M}{dt} \qquad (2.7)$$

and

$$e_2 = N_2 \frac{d\phi_M}{dt} \qquad (2.8)$$

The direction of e_1 is such to produce a current that opposes the flux change, according to Lenz's Law. If the flux varies sinusoidally such that

$$\phi_M = \phi_{\max} \sin \omega t \qquad (2.9)$$

then the corresponding induced voltage, e, linking an N-turn winding is given by

$$e = N\frac{d\phi}{dt} = \omega \cdot N \cdot \phi_{max}\cos\omega t \tag{2.10}$$

The rms value of the induced voltage is

$$E = \frac{\omega \cdot N \cdot \phi_{max}}{\sqrt{2}} = 4.44 \cdot f \cdot N \cdot \phi_{max} \tag{2.11}$$

where f is the frequency in Hertz. Equation 2.11 is known as the emf equation.

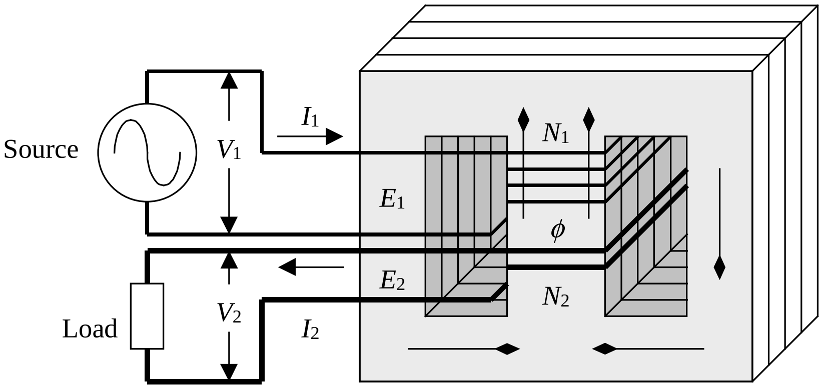


Figure 2.6. Basic Transformer Components.

<u>EXAMPLE 2.1</u>

How many turns must the primary and the secondary windings of a 220 V-110 V, 60 Hz ideal transformer have if the core flux is not allowed to exceed 5 mWb?

Solution:

For an ideal transformer with no losses,

$$E_1 \approx V_1 = 220V$$

$$E_2 \approx V_2 = 110V$$

From the emf equation, we have

$$N_1 = \frac{E_1}{4.44 \cdot f \cdot \phi_{max}}$$

$$= \frac{(220)}{(4.44)(60)(5\times10^{-3})} = 166 \ turns$$

$$N_2 = \frac{(110)}{(4.44)(60)(5\times10^{-3})} = 83 \ turns$$

2.5. The Ideal Transformer Model

Many engineering calculations can be carried out with the assumption that transformers are ideal. An ideal transformer would have windings with zero impedance and a lossless, infinite permeability core. Therefore, the efficiency would be 100%. This assumption does not introduce much error because power transformers are very nearly ideal. Their efficiencies are about 97% or better and their internal voltage drops are only about 5%.

Infinite permeability of the core would result in zero exciting current and no leakage flux. Then

$$\lambda_1 = N_1\phi, \quad \lambda_2 = N_2\phi$$

Zero resistances of the windings would result in zero voltage drops between the terminal voltages and the induced voltages (see Figure 2.7).

$$v_1 = e_1, \quad v_2 = e_2$$

For ideal transformer, Equation 2.6 can be written as:

$$a = \frac{N_1}{N_2} = \frac{v_1}{v_2} = \frac{e_1}{e_2} = \frac{i_2}{i_1} \tag{2.12}$$

Figure 2.7 shows an ideal transformer connected between a voltage source and a constant impedance load. Let V_2 and I_2 represent the secondary rms voltage and current. In phasor terms

$$I_2 = \frac{|V_2|\angle 0°}{|Z_{LD}|\angle\theta} = |I_2|\angle -\theta$$

where Z_{LD} is the impedance of the load. The input impedance seen by the source is given by

$$Z_{in} = \frac{V_1}{I_1} = \frac{aV_2}{I_2/a} = a^2\frac{V_2}{I_2} = a^2 Z_{LD}$$

Therefore, the basic relations that describe the behavior of the ideal transformer can be given as:

$$a = \frac{N_1}{N_2} = \frac{v_1}{v_2} = \frac{e_1}{e_2} = \frac{i_2}{i_1} = \sqrt{\frac{Z_{in}}{Z_{LD}}} \tag{2.13}$$

where Z_{in} is the input impedance at the primary side and Z_{LD} is the impedance that is connected to the secondary.

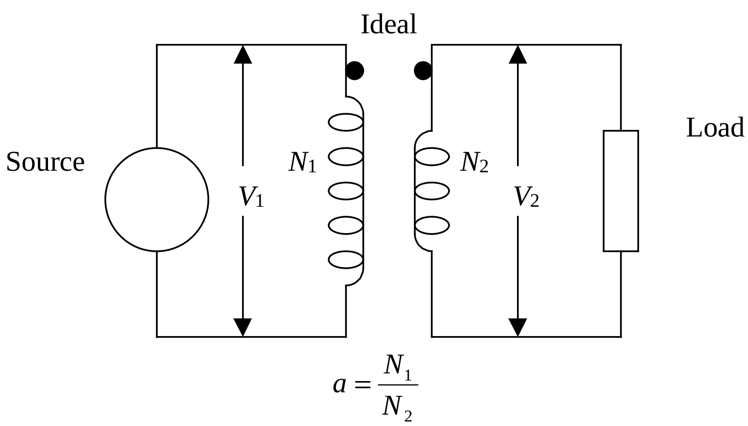


Figure 2.7 Load Connected to a Source by an Ideal Transformer.

EXAMPLE 2.2

Consider an ideal, single phase 2400 V-240 V transformer. The primary is connected to a 2200 V source and the secondary is connected to an impedance of 2 Ω ∠ 36.9°.

a) Find the secondary output current and voltage.

b) Find the primary input current.

c) Find the load impedance as seen from the primary side.

d) Find the input and output apparent powers.

e) Find the output power factor.

Solution:

a) It is standard practice to make the ratio of rated terminal voltages equal to the actual turns-ratio as:

$$a = \frac{2400V}{240V} = 10$$

$$V_2 = V_1/a = 2200V\angle 0°/10 = 220V\angle 0°$$

$$I_2 = \frac{V_2}{Z_2} = \frac{220V\angle 0°}{2\Omega\angle 36.9°} = 110A\angle -36.9°$$

b) primary current

$$I_1 = I_2/a = 110A\angle -36.9°/10 = 11A\angle -36.9°$$

c) load impedance as seen from the primary side.

$$Z_{in} = \frac{V_1}{I_1} = \frac{2200V\angle 0°}{11A\angle -36.9°} = 200\Omega\angle 36.9°$$

d) input power:

$$S_1 = V_1 I_1^* = (2200V\angle 0°)(11A\angle 36.9°) = 24.2kVA\angle 36.9°$$

output power:

$$S_2 = V_2 I_2^* = (220V\angle 0°)(110A\angle 36.9°) = 24.2kVA\angle 36.9°$$

e) power factor:

$$PF = \cos\phi = \cos(36.9°) = 0.8 \quad lagging$$

2.7. Non-ideal Transformer and the Exact Equivalent Model

The non-ideal transformer has hysteresis and eddy-current (or core) losses, and resistive (or I^2R) losses in the primary and secondary windings. Furthermore, the core of a non-ideal transformer is not perfectly permeable, and the transformer core requires a finite magnetic motive force (mmf) for its magnetization. Also, not all fluxes link with the primary and the secondary windings simultaneously because of leakage around the windings. This is called leakage flux.

An equivalent circuit of the non-ideal transformer is shown in Figure 2.9. The dot markings indicate terminals of corresponding polarity in the sense that both windings encircle the core in the same direction if we begin at the dots.

By using Equation 2.13, the ideal transformer may be removed from Figure 2.9, and the entire exact equivalent circuit may be referred either to the primary, as shown in Figure 2.10, or to the secondary, as shown in Figure 2.11.

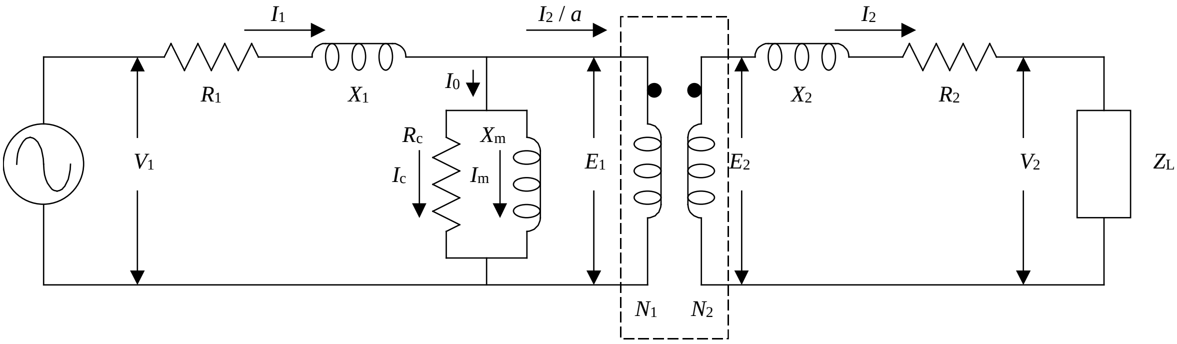


Figure 2.9. Exact Equivalent Circuit of a Non-Ideal Transformer.

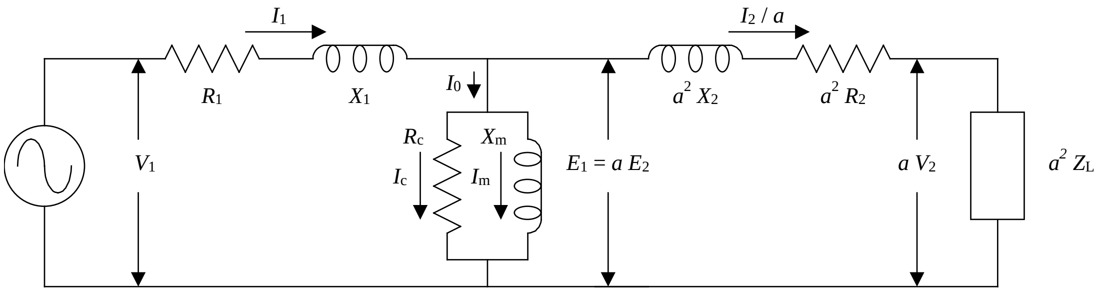


Figure 2.10. Exact Equivalent Circuit as Referred to the Primary.

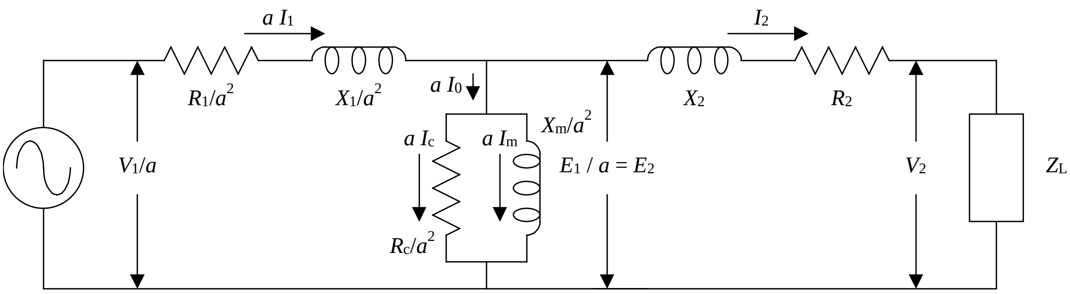


Figure 2.11. Exact Equivalent Circuit as Referred to the Secondary.

The various symbols used in Figures 2.10 and 2.11 are:

a	turns ratio
E_1, E_2	primary and secondary induced voltages
V_1, V_2	primary and secondary terminal voltages
I_1, I_2	primary and secondary currents
I_ϕ, I_0	no load current

r_1, x_1	primary winding resistance and reactance
r_2, x_2	secondary winding resistance and reactance
I_m, X_m	magnetizing current and reactance
I_c, R_c	core loss current and resistance

The major use of the exact equivalent circuit of a transformer is to determine the characteristics such as voltage regulation and efficiency.

A phasor diagram for the circuit of Figure 2.11, for lagging power factor can be obtained by using the following equations:

$$I_0 = I_c + I_m \tag{2.14}$$

where I_c is in phase with E_1 and I_m lags E_1 by 90°.

$$I_1 = \frac{I_2}{a} + I_0 \tag{2.15}$$

$$E_2 = V_2 + I_2(r_2 + jx_2) \tag{2.16}$$

$$V_1 = aE_2 + I_1(r_2 + jx_2) \tag{2.17}$$

Based on Equations 2.14 through 2.17 and assuming a zero degree reference angle for V_2, the phasor diagram is shown in Figure 2.12 for the exact equivalent circuit model of a transformer.

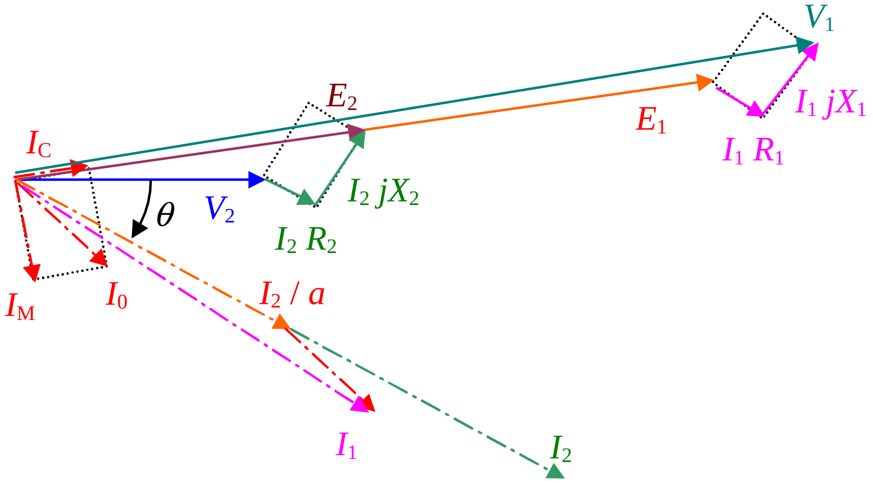


Figure 2.12. RMS Phasor Diagram for the Exact Equivalent Circuit Model of a Transformer.

2.8. The Approximate Transformer Circuit Model

The exact equivalent transformer model is more accurate than is necessary for most engineering calculations. A simpler model, the approximate circuit, is most frequently used. The approximate circuit of the transformer referred to the primary side is shown in Figure 2.13.

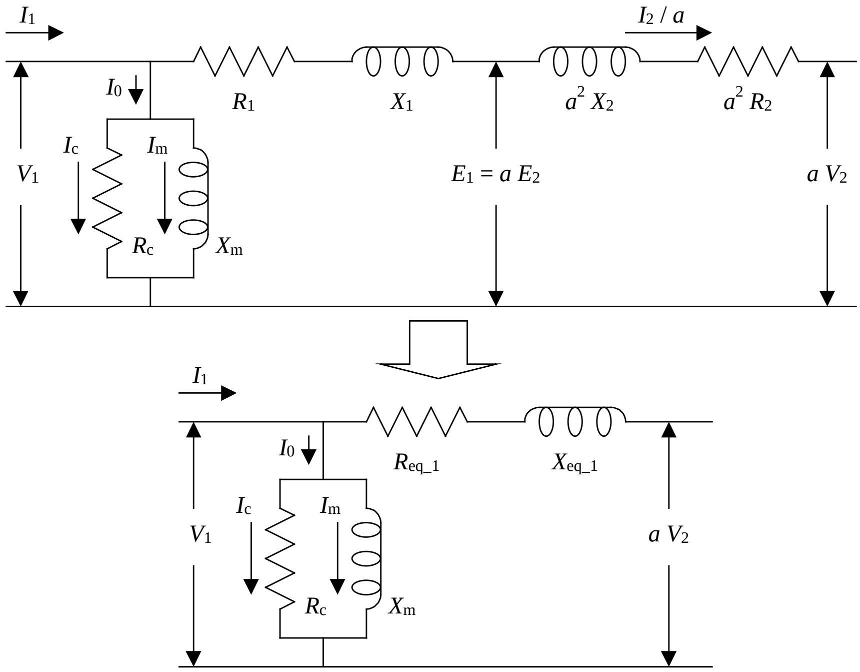


Figure 2.13. Approximate Circuit Model of a Transformer Referred to the Primary.

The rational for these approximate equivalent circuits is that the voltage in the primary series impedance ($r_1 + jx_1$) is small, even at full load. Also, the no load current (I_0) is so small that its effect on the voltage drop in the primary series impedance is negligible. Therefore, it matters little if the shunt branch of R_c in parallel with X_m is connected before the primary series impedance or after it. The core loss and magnetizing currents are not greatly affected by the move. Connecting the shunt components right at the input terminals has the great advantage of permitting the two series impedance to be combined into one complex impedance.

The value of this equivalent impedance of a particular transformer depends, of course, on whether the model used is referred to the primary or secondary. Figure 2.14 shows the approximate equivalent circuit of transformer referred to the secondary side.

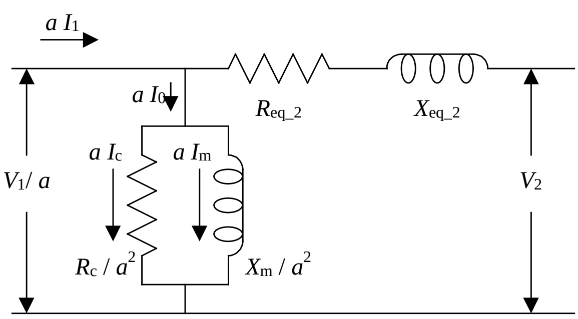


Figure 2.14. Approximate Circuit Model of a Transformer Referred to the Secondary.

If the circuit is referred to the primary as shown in Figure 2.13,

$$Z_{eq\_1} \equiv R_{eq\_1} + jX_{er\_1} = \left(r_1 + a^2 r_2\right) + j\left(x_1 + a^2 x_2\right) \tag{2.18}$$

If the circuit is referred to the secondary as shown in Figure 2.14,

$$Z_{eq\_2} \equiv R_{eq\_2} + jX_{er\_2} = \left(r_1/a^2 + r_2\right) + j\left(x_1/a^2 + x_2\right) \tag{2.19}$$

2.9. Transformer Characteristics

The major use of the exact and approximate equivalent circuits of a transformer is in determining its characteristics. The characteristics of most interest to power engineers are voltage regulation and efficiency.

2.9.1. Voltage Regulation

Voltage regulation is a measure of the change in the terminal voltage of the transformer with respect to loading. Therefore the voltage regulation is defined as:

$$percent \quad regulation \quad = \frac{\left|V_{2 \quad no \quad load}\right| - \left|V_{2 \quad load}\right|}{\left|V_{2 \quad load}\right|} \times 100 \tag{2.20}$$

With reference of Figure 2.14,

$$\frac{V_1}{a} = V_2 + I_2 Z_{eq\_2} \tag{2.21}$$

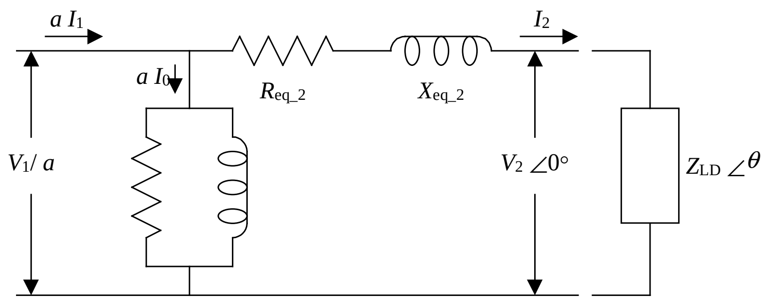
and at no load ($I_2 = 0$)

$$\frac{V_1}{a} = V_{2 \quad no \quad load}$$

Equation 2.20 can be written as:

$$percent \quad regulation \quad = \frac{\left|V_1/a\right| - \left|V_2\right|}{\left|V_2\right|} \times 100 \tag{2.22}$$

It is clear from Equation 2.21 that the terminal voltage V_1 is load dependent. Examples of transformer voltage regulation are shown in Figure 2.15.



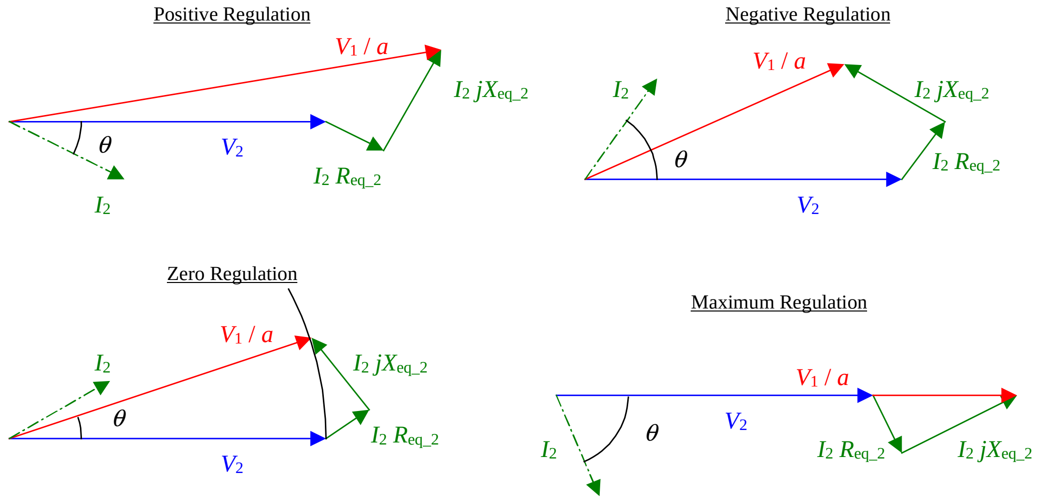


Figure 2.15. Examples of Transformer Voltage Regulation.

2.9.2. Transformer Efficiency

Efficiency of a transformer is defined as follows:

$$\eta = \frac{output \quad power}{input \quad power} = \frac{P_2}{P_1} \tag{2.23}$$

For a non-ideal transformer, the output power is less than the input power because of losses. These losses are the winding or I^2R loss (copper losses) and the core loss (hysteresis and eddy-current losses).

Thus, in terms of the total losses, P_{losses}, Equation 2.22 may be expressed as:

$$\eta = \frac{P_1 - P_{losses}}{P_1} = \frac{P_2}{P_2 + P_{losses}} = \frac{P_2}{P_2 + P_{copper} + P_{core}} \tag{2.24}$$

Obviously, the winding or copper loss is load dependent, whereas the core loss is constant and almost independent of the load on the transformer.

The efficiency can also be obtained by using the per-unit system. Dividing both the numerator and denominator by the rated power base of the transformer.

$$\eta = \frac{P_2 / S_{Base}}{P_2 / S_{Base} + P_{copper} / S_{Base} + P_{core} / S_{Base}} \tag{2.25}$$

or

$$\eta = \frac{\left(\left|S_2\right| / S_{Base}\right)\cos\phi}{\left(\left|S_2\right| / S_{Base}\right)\cos\phi + P_{copper} / S_{Base} + P_{core} / S_{Base}} \tag{2.26}$$

In efficiency calculations, it is usually assumed that the secondary voltage is at rated value. With constant terminal voltage, the magnitude of the secondary current is proportional to the volt-ampere load (apparent power of the load). The winding or copper loss, which is a function of the current, is therefore proportional to the square of the load. As a result,

$$P_{copper} = P_{cu} = \left(\frac{\left|S_2\right|}{S_{Base}}\right)^2 \cdot P_{cu \quad full \quad load}$$

or in per unit

$$P_{cu-pu} = \left|S_{2-pu}\right|^2 \cdot P_{cu-pu \quad full \quad load}$$

Since

$$\frac{\left|I_{2 \quad full \quad load}\right|}{I_{Base}} = 1.0\, pu$$

then

$$P_{cu-pu \quad full \quad load} = I_{2 \quad full \quad load} R_{eq-pu} = R_{eq-pu}$$

$$P_{cu-pu} = \left|S_{2-pu}\right|^2 \cdot R_{eq-pu}$$

The per-unit form of the efficiency expression becomes

$$\eta = \frac{\left|S_{2-pu}\right|\cos\phi}{\left|S_{2-pu}\right|\cos\phi + P_{core-pu} + \left|S_{2-pu}\right|^2 R_{eq-pu}} \tag{2.27}$$

At full load, $S_{2\text{-pu}}$ = 1.0pu; therefore, the efficiency at full load is

$$\eta_{FL} = \frac{\cos\phi}{\cos\phi + P_{core-pu} + R_{eq-pu}} \tag{2.28}$$

where $\cos\phi$ is the load power factor.

2.9.3. Maximum Efficiency

It is often desirable to design a transformer to have maximum efficiency at some particular loading. When the voltage and frequency are both constant, the core losses are nearly constant. Therefore, the losses may then be divided into two categories:

- Constant losses such as core losses, P_c.

- Variable losses such as winding losses, I^2R

Equation 2.24 can be written as:

$$\eta = \frac{k \cdot V_2 I_2}{k \cdot V_2 I_2 + P_c + I_2^2 R_{eq\_2}} \tag{2.29}$$

where kV_2I_2 represents the output power and k represents a constant. For example, in a single-phase transformer, the load's power factor. The efficiency will be maximum or minimum when

$$\frac{\partial \eta}{\partial I_2} = 0, \qquad \text{or when}$$

$$\frac{\left(k \cdot V_2 I_2 + P_c + I_2^2 R_{eq\_2}\right) \cdot \left(k \cdot V_2\right) - \left(k \cdot V_2 I_2\right) \cdot \left(k \cdot V_2 + 2 I_2 R_{eq\_2}\right)}{\left(k \cdot V_2 I_2 + P_c + I_2^2 R_{eq\_2}\right)^2} = 0$$

$$P_c - I_2^2 R_{eq\_2} = 0$$

$$P_c = I_2^2 R_{eq\_2}$$

or

$$P_c = P_{cu} \tag{2.30}$$

Thus maximum efficiency occurs when the variable losses equal the constant losses.

EXAMPLE 2.3

A 10 kVA, 2400V/240V, single-phase transformer has the following resistances and leakage reactances:

$r_1 = 3.0\Omega$ $\quad x_1 = 15.0\Omega$ $\quad r_2 = 0.03\Omega$ $\quad x_2 = 0.15\Omega$

Use the approximate circuit model to find the voltage regulation when the load power factor is: a) 0.8 lagging, and b) 0.8 leading.

Solution

a) Calculate the equivalent impedance.

$$a = \frac{2400V}{240V} = 10$$

$$Z_{eq\_2} = \left(\frac{r_1}{a^2} + r_2\right) + j\left(\frac{x_1}{a^2} + x_2\right)$$

$$= \left(\frac{3\Omega}{100} + 0.03\Omega\right) + j\left(\frac{15\Omega}{100} + 0.15\Omega\right) = (0.06 + j0.30)\Omega = 0.306\Omega\angle 78.7°$$

At 0.8 PF lagging, and choosing a zero degree voltage reference at the secondary terminal, the full load secondary current is:

$\phi = \arccos(0.8) = 36.9°$

$$I_2 = \left(\frac{S_{Ld}}{V_2}\right)^* = \left(\frac{10kVA\angle 36.9°}{240V\angle 0°}\right)^* = 41.7\angle -36.9°$$

By using the circuit in Figure 2.14, we find:

$$\frac{V_1}{a} = V_2 + I_2 Z_{eq\_2}$$
$$= 240V\angle 0° + (41.7A\angle -36.9°)(0.306\Omega\angle 78.7°)$$
$$= (249.5 + j8.51)V = 249.7V\angle 1.95°$$

$$Voltage \quad regulation \quad = \frac{|V_1/a| - |V_2|}{|V_2|}$$
$$= \frac{249.7 - 240}{240} = 0.0404 = 4.04\%$$

b) At 0.8 PF leading, the full load secondary current is:

$$\frac{V_1}{a} = 240V\angle 0° + (41.7A\angle +36.9°)(0.306\Omega\angle 78.7°)$$
$$= (234.5 + j11.51)V = 234.8V\angle 2.81°$$
$$Voltage \quad regulation \quad = \frac{234.8 - 240}{240} = -0.022 = -2.2\%$$

EXAMPLE 2.4

Use the per-unit system to find the voltage regulation in Example 2.3.

Solution:

Choose the base values as follows:

$$S_{1\phi-Base} = 10kVA$$
$$V_{1\_Base} = 2400V \qquad V_{2\_Base} = 240V$$
$$I_{2\_Base} = \frac{S_{1\phi-Base}}{V_{2\_Base}} = \frac{10kVA}{240V} = 41.7A$$
$$Z_{2\_Base} = \frac{V_{2\_Base}}{I_{2\_Base}} = \frac{240V}{41.7A} = 5.756\Omega$$

a) The per-unit load current at full load at 0.8 PF lagging is:

$$I_{2-pu} = 1.0\angle -36.9°$$

$$Z_{2\_eq-pu} = \frac{Z_{2\_eq}}{Z_{2\_Base}} = \frac{0.306\Omega\angle 78.7°}{5.756\Omega} = 0.05315\angle 78.7°$$

$$V_{2-pu} = 1.0\angle 0°$$

$$V_{1-pu} = (1.0\angle 0) + (1.0\angle -36.9°)(0.05315\angle 78.7°) = 1.0 + 0.05315\angle 41.8° = 1.0404\angle 2.0°$$

$$Voltage \quad regulation \quad = \frac{1.0404 - 1.0}{1.0} = 0.0404 = 4.04\%$$

b)

$$I_{2-pu} = 1.0\angle +36.9°$$

$$V_{1-pu} = (1.0\angle 0) + (1.0\angle 36.9°)(0.05315\angle 78.7°) = 1.0 + 0.05315\angle 115.6° = 0.978\angle 2.8°$$

$$Voltage \quad regulation \quad = \frac{0.978 - 1.0}{1.0} = -0.022 = -2.2\%$$

EXAMPLE 2.5

A 2400V/120V, 300 kVA, single-phase transformer has a core loss at rated voltage of 2.7 kW. If its equivalent resistance is 1.4%, find the efficiency of this transformer for a load power factor of 0.9: a) at full load and b) at half load. What is the load at maximum efficiency?

Solution:

a) The per-unit core loss is given by:

$$S_{1\phi-Base} = 300kVA$$

$$P_c = 2.7kW$$

$$P_{c-pu} = \frac{P_c}{S_{1\phi-Base}}$$

$$= \frac{(2.7kW)}{(300kVA)} = 0.009\,pu$$

The full load winding losses (copper losses) are:

$$I_{2-FL} = 1.0\,pu$$

$$R_{eq\_2} = 0.014\,pu$$

$$P_{cu} = (I_{2-FL})^2 R_{eq\_2}$$

$$= (1.0)^2(0.014) = 0.014\,pu$$

Then Equation 2.28 can be applied for a 0.9 power factor.

$$\eta_{FL} = \frac{(0.9)}{(0.9) + (0.009) + (0.014)} = 0.975 \qquad or \quad 97.5\%$$

b) For half load, Equation 2.27 provides:

$$\frac{|S_{Ld}|}{S_{1\phi-Base}} = 0.5\,pu$$

$$\eta_{\frac{1}{2}Ld} = \frac{(0.5)(0.9)}{(0.5)(0.9) + (0.009) + (0.5)^2(0.014)} = 0.973 \quad or \quad 97.3\%$$

Maximum efficiency occurs when

$$\left(\frac{|S_{Ld}|}{S_{1\phi-Base}}\right)^2 R_{eq-pu} = P_{c-pu}$$

Then for maximum efficiency

$$S_{Ld-pu} = \sqrt{\frac{P_{c-pu}}{R_{eq-pu}}} = \sqrt{\frac{0.009\,pu}{0.014\,pu}} = 0.802\,pu$$

In other words, this transformer has its maximum efficiency at about 80% of its full load.

2.10. The Three-Phase Transformer

Three-phase power may be transformed by a bank of single-phase transformers or by a single three-phase transformer. A three-phase transformer is essentially three transformers wound on a common core. The geometry of the core is such that the fluxes of the phases share common paths. As a result, the volume of iron is less than that of three single-phase units of the same total rating.

These are some advantages and disadvantages in using a three-phase transformer instead of a bank of single-phase transformers. The advantages are:

- It takes up less space
- It is less expensive.
- In involves less external wiring.
- It has slightly better efficiency.

The disadvantages are:

- It does not provide the flexibility of a set of single-phase units. For example, one single-phase transformer in a bank may have a higher power rating than others, to serve an unbalanced load.
- In case of failure of a single-phase unit serving in a bank, only that one unit needs to be replaced. However, it is most likely that damage within a three-phase unit will require complete removal from service all three phases and a replacement of the complete unit.

As a result, three-phase banks of single-phase transformers are seldom used in new installations except in distribution circuits serving a combination of single and three-phase loads.

In three-phase transformation, the primary and secondary windings may be connected independently in either delta or wye. The possible combinations are listed below and are shown in Figure 2.16.

- delta-delta
- delta-wye
- wye-delta
- wye-wye

The wye-wye connection is to be avoided unless a very solid neutral connection is made between the primary and the power source. If a neutral is not provided, the phase voltages tend to become severely unbalanced when the load is unbalanced. There are also troubles with third harmonics. These problems do not exist when one of the sets of windings is in delta configuration.

When a wye-delta or delta-wye connection is used, the wye is preferably on the high-voltage side in transmission systems, and the neutral is grounded. The transformer insulation may thus be designed for $1/\sqrt{3}$ times the line voltage, rather than for the full line voltage. Sometimes it is necessary to have the wye connection on the low-voltage side, if the neutral is required for the low-voltage circuit.

Wye-delta and delta-wye connections result in 30° phase displacement between primary and secondary line voltages. It is standard practice in the United States to connect these transformers in such a way that the lower voltages lag the higher voltages by 30°.

The exact and the approximate equivalent circuit models of the transformer described in the last sections can be applied to the three-phase transformer to represent just one of the three phases.

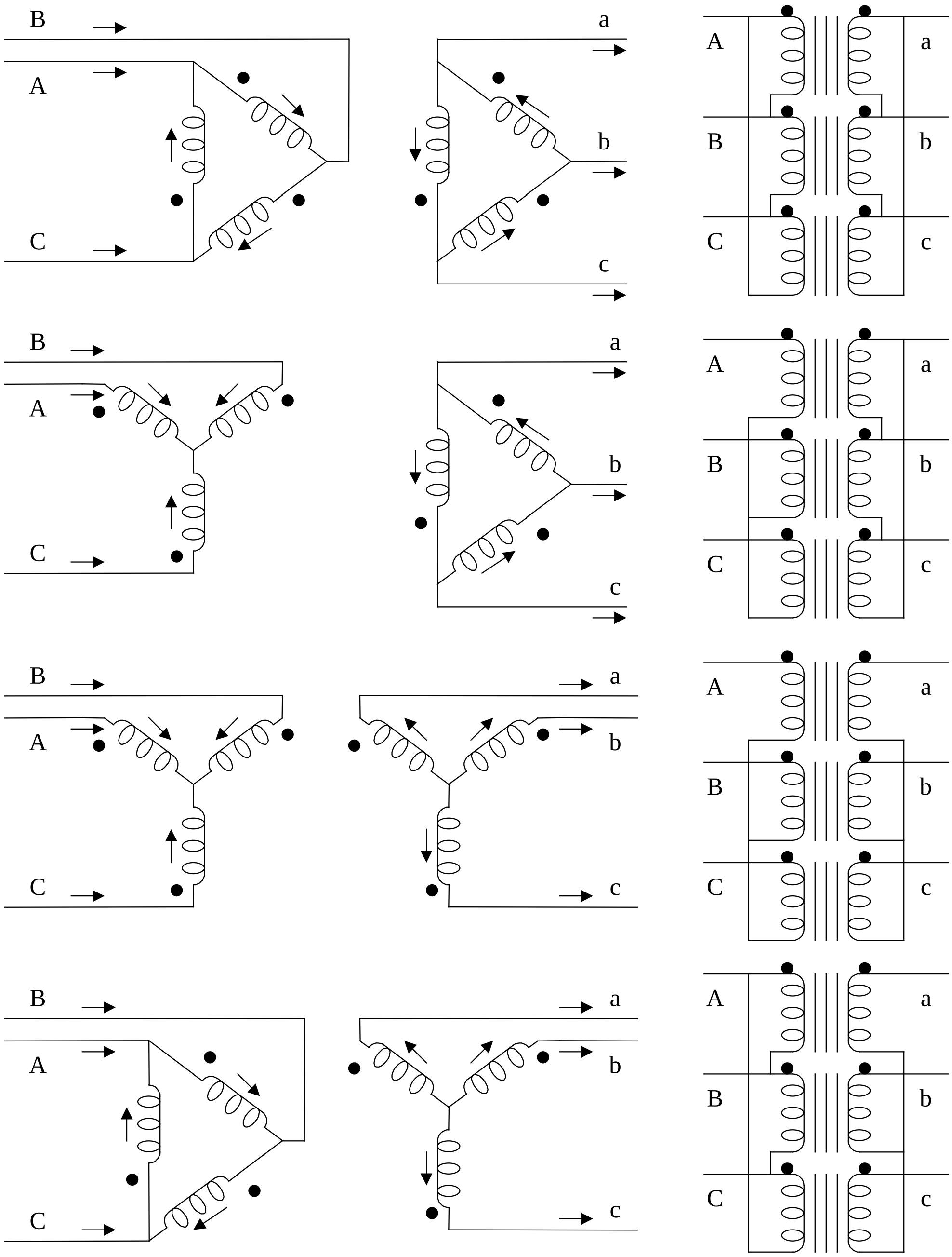


Figure 2.16. Basic Transformer Connections for Three-Phase Units.

EXAMPLE 2.6

What should be the ratings (voltages and currents) and turns ratio of a three-phase transformer to transform 10 MVA from 230 kV to 4160 V, if the transformer is to be connected: a) wye-delta, b) delta-wye, and c) delta-delta?

Solution

For both delta and wye connections, the line currents can be obtained as:

$$I_{L1} = \frac{S_{3\phi}}{\sqrt{3}\cdot V_{L1}} = \frac{\left(10\times 10^6 VA\right)}{\sqrt{3}\left(230\times 10^3 V\right)} = 25.1A$$

$$I_{L2} = \frac{S_{3\phi}}{\sqrt{3}\cdot V_{L2}} = \frac{\left(10\times 10^6 VA\right)}{\sqrt{3}\left(4160V\right)} = 1388A$$

The rated power per phase:

$$S_{1\phi} = S_{3\phi}/3 = 10MVA/3 = 3333.3kVA$$

a) wye-delta connected:

$$I_{p\_1} = I_{L1} = 25.1A$$

$$I_{p\_2} = I_{L2}/\sqrt{3} = \left(1388A\right)/\sqrt{3} = 801A$$

$$V_{p\_1} = V_{L1}/\sqrt{3} = \left(230kV\right)/\sqrt{3} = 132.8kV$$

$$V_{p\_2} = V_{L2} = 4160V$$

$$a = \frac{V_{p\_1}}{V_{p\_2}} = \frac{132.8kV}{4160V} = 31.9$$

b) delta-wye connected:

$$I_{p\_1} = I_{L1}/\sqrt{3} = \left(25.1A\right)/\sqrt{3} = 14.5A$$

$$I_{p\_2} = I_{L2} = 1388A$$

$$V_{p\_1} = V_{L1} = 230kV$$

$$V_{p\_2} = V_{L2}/\sqrt{3} = \left(4160V\right)/\sqrt{3} = 2400V$$

$$a = \frac{V_{p\_1}}{V_{p\_2}} = \frac{230kV}{2400V} = 95.8$$

c) delta-delta connected:

$$I_{p\_1} = I_{L1}/\sqrt{3} = \left(25.1A\right)/\sqrt{3} = 14.5A$$

$$I_{p\_2} = I_{L2}/\sqrt{3} = \left(1388A\right)/\sqrt{3} = 801A$$

$$V_{p\_1} = V_{L1} = 230kV$$

$$V_{p\_2} = V_{L2} = 4160V$$

$$a = \frac{V_{p\_1}}{V_{p\_2}} = \frac{230kV}{4160V} = 55.3$$

EXAMPLE 2.7

A 7200V/208V, 50kVA, three-phase distribution transformer is connected delta-wye. The transformer has 1.2% resistance and 5% reactance. Find the voltage regulation at full load, 0.8 power factor lagging.

Solution:

Delta Connection:

$$V_{p1} = V_{L1} = 7200V$$

$$I_{L1} = \frac{50kVA}{\sqrt{3}(7200V)} = 4.01A$$

$$I_{p1} = \frac{I_{L1}}{\sqrt{3}} = \frac{4.01A}{\sqrt{3}} = 2.32A$$

Wye Connection:

$$V_{p2} = \frac{V_{L2}}{\sqrt{3}} = \frac{208V}{\sqrt{3}} = 120V$$

$$I_{L2} = \frac{50kVA}{\sqrt{3}(208V)} = 138.8A$$

$$I_{p2} = I_{L2}$$

$$a = \frac{V_{1p}}{V_{2p}} = \frac{7200V}{120V} = 60$$

The base impedance in the secondary side:

$$Z_{2\_Base} = \frac{V_{2p}}{I_{2p}} = \frac{120V}{138.8A} = 0.865\Omega$$

$$Z_{eq\_pu} = R_{eq\_pu} + jX_{eq\_pu} = 0.012 + j0.050\,pu = 0.0514\angle 76.5°$$

$$Z_{eq\_2} = Z_{eq\_pu} \cdot Z_{2\_Base} = 0.0514\,pu\angle 76.5°(0.865\Omega) = 0.0445\Omega\angle 76.5°$$

At full load, 0.8 power factor, lagging

$$I_2 = 138.8A\angle -36.9°$$

$$\frac{V_1}{a} = V_2 + I_2 Z_{eq\_2}$$

Note that all the quantities in this equation have to be phase quantities. The no-load secondary phase voltage is:

$$\frac{V_1}{a} = 120V\angle 0° + (138.8A\angle -36.9°)(0.0445\angle 76.5°)$$

$$= 124.8 + j3.94 = 124.8V\angle 1.8°$$

The no-load secondary line voltage:

$$\frac{V_{1\_LL}}{a} = \sqrt{3}(124.8V)$$

Regulation:

$$VR = \frac{\sqrt{3}(124.8V) - 208V}{208V} = \frac{124.8V - 120V}{120V} = 0.04 = 4\%$$

The problem can also be solved by using the per-unit method as:

$$VR = \frac{|V_{1-pu}| - |V_{2-pu}|}{|V_{2-pu}|}$$

$$V_{1-pu} = V_{2-pu} + I_{2-pu} Z_{eq-pu} = 1\angle 0° + (1\angle -36.9°)(0.0514\angle 76.5°)$$

$$= 1.040 + j0.0328 = 1.04\angle 1.8°$$

$$VR = \frac{1.04 - 1.0}{1.0} = 0.04 = 4\%$$

EXAMPLE 2.8

If the core loss of the transformer in Example 2.7 is 1kW, find the efficiency of this transformer at full load and 0.8 power factor.

Solution:

$$S_{Base} = 50kVA$$

$$P_c = \frac{1kW}{50kVA} = 0.02\,pu$$

Applying Equation 2.28

$$\eta_{FL} = \frac{0.8}{0.8 + 0.02 + 0.012} = 0.962 = 96.2\%$$

2.11. Measuring Transformer Quantities

2.11.1. Single-Phase Transformer

The parameters of the approximate equivalent circuit model of a transformer may be determined by two tests: a) open-circuit test and b) short-circuit test.

OPEN-CIRCUIT TEST

The open circuit test is conducted by applying rated voltage at rated frequency to one of the windings, with the other windings open circuited. The input power and current are measured. For reasons of safety and convenience, the measurements are made on the low-voltage (LV) side of the transformer.

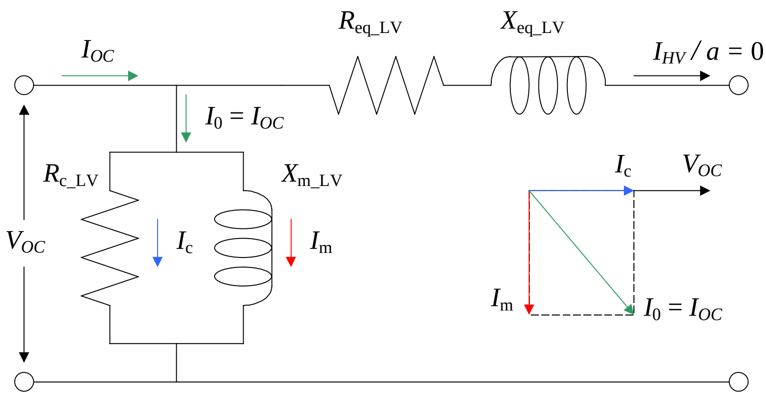


Figure 2.17. Equivalent Circuit of the Open-Circuit Test.

The equivalent circuit for the open-circuit test is as shown in Figure 1.17. Since the high voltage (HV) side is open, the input current is equal to the no load current or exciting current (I_0), and is quite small. The voltage drops in the primary leakage reactance and winding resistance may be neglected and so may the primary loss ($I_1^2 r_1$). The input power is almost equal to the core loss at rated voltage and frequency.

$$P_{oc} = P_{core} = \frac{V_{oc}^2}{R_c} \quad (2.31)$$

$$P_{core} = V_{oc} I_c \quad (2.32)$$

$$\cos\theta_{oc} = \frac{P_{oc}}{V_{oc} I_{oc}} \quad (2.33)$$

θ_{oc} is the angle by which I_{o_LV} lags V_{oc}. The core loss current, I_c is in phase with V_{oc} while I_m lags V_{oc} by 90°. Then

$$I_c = I_{oc} \cos\theta_{oc} \quad (2.34)$$

$$I_m = I_{oc} \sin\theta_{oc} \quad (2.35)$$

and

$$I_0 = I_{oc} = \sqrt{I_c^2 + I_m^2} \quad (2.36)$$

The core-loss current, I_c, may be found from Equation 2.32, then R_{c_LV} may be calculated by Equation 2.31 or by

$$R_{c\_LV} = \frac{V_{oc}}{I_c} \quad (2.37)$$

The magnetization current I_m is given by Equation 2.35 or may be found from I_{oc} and I_c using Equation 2.36. Then

$$X_{m\_LV} = \frac{V_{oc}}{I_m} \quad (2.38)$$

SHORT-CIRCUIT TEST

The short-circuit test is used to determine the equivalent series resistance and reactance. One winding is shorted at its terminals, and the other winding is connected through proper meters to a variable, low-voltage, high-current source of rated frequency. The source voltage is increased until the current into the transformer reaches rated value. To avoid unnecessary high currents, the short-circuit measurements are made on the high-voltage side of the transformer. The test circuit with the effective equivalent circuit is shown in Figure 2.18.

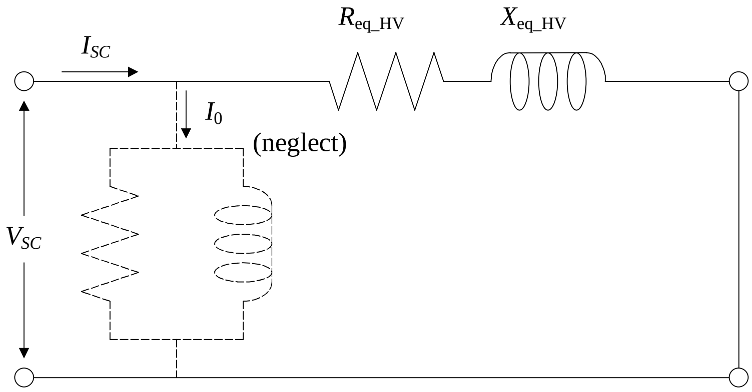


Figure 2.18. Equivalent Circuit of the Short-Circuit Test.

Neglecting I_0, the input power during this test is consumed in the equivalent resistance referred to the primary or high-voltage side, R_{eq_HV}. Then

$$P_{sc} = I_{sc}^2 R_{eq\_HV}$$

and

$$R_{eq\_HV} = \frac{P_{sc}}{I_{sc}^2} \tag{2.39}$$

Since rated current is used, the winding or copper loss during the short-circuit test is equal to the full-load copper loss:

$$P_{sc} = P_{copper}\left(I_1 = full \quad load\right)$$

From Figure 2.18,

$$\begin{aligned} \left|Z_{eq\_HV}\right| &= \frac{V_{sc}}{I_{sc}} \\ &= \sqrt{R_{eq\_HV}^2 + X_{eq\_HV}^2} \end{aligned} \tag{2.40}$$

Then, having found R_{eq_HV} from Equation 2.39,

$$X_{eq\_HV} = \sqrt{Z_{eq\_HV}^2 - R_{eq\_HV}^2} \tag{2.41}$$

or

$$X_{eq\_HV} = \left|Z_{eq\_HV}^2\right| \sin\theta_{sc} \tag{2.42}$$

where

$$\theta_{sc} = \arccos\left(\frac{P_{sc}}{V_{sc} I_{sc}}\right)$$

2.11.2. Three Phase Transformer

All the measured quantities in a three-phase transformer are line voltages, line currents, and total power. The impedances must be calculated on a phase basis.

<u>OPEN-CIRCUIT TEST</u>

Measurements are made on the low-voltage side and then converted to phase quantities. It is necessary to know whether the low-voltage windings are delta or wye connected. The test is made at rated low voltage and rated frequency.

Delta-Connected Low-Voltage Winding	Wye-Connected Low-Voltage Winding
$P_{\phi\_oc} = \frac{P_{oc}}{3}$ $V_{p\_oc} = V_{oc}$ $I_{p\_oc} = \frac{I_{oc}}{\sqrt{3}}$ $P_{3\phi\_core} = P_{oc}$ $R_{c\_LV} = \frac{3 \cdot V_{oc}^2}{P_{oc}}$ $\cos\theta_{oc} = \frac{P_{oc}}{\sqrt{3}(V_{oc})I_{oc}}$ $I_m = \frac{I_{oc}}{\sqrt{3}}\sin\theta_{oc}$ $X_{m\_LV} = \frac{V_{p\_oc}}{I_m}$	$P_{\phi\_oc} = \frac{P_{oc}}{3}$ $V_{p\_oc} = \frac{V_{oc}}{\sqrt{3}}$ $I_{p\_oc} = I_{oc}$ $P_{3\phi\_core} = P_{oc}$ $R_{c\_LV} = \frac{V_{oc}^2}{P_{oc}}$ $\cos\theta_{oc} = \frac{P_{oc}}{\sqrt{3}(V_{oc})I_{oc}}$ $I_m = I_{oc}\sin\theta_{oc}$ $X_{m\_LV} = \frac{V_{p\_oc}}{I_m}$

<u>SHORT-CIRCUIT TEST</u>

The low-voltage terminals are shorted together. The voltage applied to the high-voltage terminals is adjusted so that rated current flows. The frequency of the voltage source is the rated frequency of the transformer. The connection (delta or wye) of the high-voltage windings should be known.

Delta-Connected High-Voltage Winding	Wye-Connected High-Voltage Winding
$P_{\phi\_sc} = \frac{P_{sc}}{3}$	$P_{\phi\_sc} = \frac{P_{sc}}{3}$
$V_{p\_sc} = V_{sc}$	$V_{p\_sc} = \frac{V_{sc}}{\sqrt{3}}$
$I_{p\_sc} = \frac{I_{sc}}{\sqrt{3}}$	$I_{p\_sc} = I_{sc}$
$P_{3\phi\_copper} = P_{sc}$	$P_{3\phi\_copper} = P_{sc}$
$Z_{eq\_HV} = \frac{\sqrt{3} \cdot V_{sc}}{I_{sc}}$	$Z_{eq\_HV} = \frac{V_{sc}}{\sqrt{3} \cdot I_{sc}}$
$R_{eq\_HV} = \frac{P_{sc}}{(I_{sc})^2}$	$R_{eq\_HV} = \frac{P_{sc}}{3 \cdot (I_{sc})^2}$
$X_{eq\_HV} = \sqrt{\lvert Z_{eq\_HV} \rvert^2 - \lvert R_{eq\_HV} \rvert^2}$	$X_{eq\_HV} = \sqrt{\lvert Z_{eq\_HV} \rvert^2 - \lvert R_{eq\_HV} \rvert^2}$

EXAMPLE 2.9

Consider a 50 kVA, 7200V-208V, three-phase, 60 Hz, delta-wye transformer. The open- and short-circuit tests are as follows:

P_{oc} = 500 W	P_{sc} = 600 W
I_{oc} = 8.0 A	I_{sc} = 4.01 A
V_{oc} = 208 V	V_{sc} = 370 V

Find:

a) The parameters of the approximate equivalent circuit referred to the high-voltage winding, then draw the circuit with its parameters.

b) The parameters of the approximate equivalent circuit referred to the low-voltage winding.

c) The per-unit equivalent series resistance and reactance.

d) The voltage regulation at full load, 0.8 power-factor lagging.

e) The efficiency at full load and at 0.8 power factor. What is the efficiency at 25% load and the same power factor?

Solution:

a) From the open circuit test, the total core loss = P_{oc} = 500 W.

The low voltage side is wye connected. Then:

$$P_{\phi\_oc} = \frac{500W}{3} = 166.7W$$

$$V_{p\_oc} = \frac{208V}{\sqrt{3}} = 120V$$

$$I_{p\_oc} = I_0 = I_{oc} = 8.0A$$

$$R_{c\_LV} = \frac{(208V)^2}{500W} = 86.5\Omega$$

$$\theta_{oc} = \arccos\left(\frac{500W}{\sqrt{3}\cdot(208V)(8.0A)}\right) = 80.0°$$

$$I_m = (8.0A)\sin(80.0°) = 7.88A$$

$$X_{m\_LV} = \frac{120V}{7.88A} = 15.23\Omega$$

From the short-circuit test, the full load winding losses = P_{sc} = 600W.

The high-voltage side is delta connected.

$$R_{eq\_HV} = \frac{600W}{(4.01A)^2} = 37.3\Omega$$

$$Z_{eq\_HV} = \frac{\sqrt{3}\cdot(370V)}{4.01A} = 159.8\Omega$$

$$X_{eq\_HV} = \sqrt{(159.8\Omega)^2 - (37.3\Omega)^2} = 155.4\Omega$$

Referring R_c and X_m to the high voltage side:

$$a = \frac{N_1}{N_2} = \frac{V_{1\_p}}{V_{2\_p}} = \frac{7200V}{120V} = 60$$

$$R_{c\_HV} = R_{c\_LV}\cdot a^2 = 86.5\Omega\cdot(60)^2 = 311k\Omega$$

$$X_{m\_HV} = 15.23\Omega\cdot(60)^2 = 54.8k\Omega$$

The equivalent circuit referred to the high voltage side is shown in Figure 2.19.

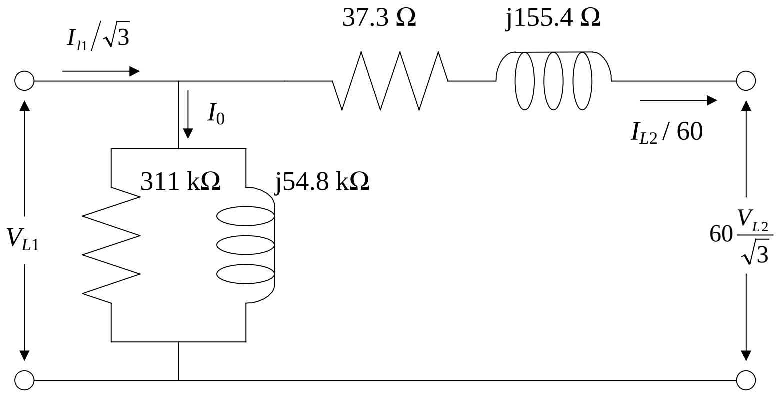


Figure 2.19. Equivalent Circuit of the Transformer in Example 2.9.

b) The parameters referred to the low voltage side are:

$$R_{c\_LV} = 86.5\Omega$$

$$X_{c\_LV} = 15.23\Omega$$

$$R_{eq\_LV} = \frac{37.3\Omega}{(60)^2} = 0.01036\Omega$$

$$X_{eq\_LV} = \frac{155.4\Omega}{(60)^2} = 0.0432\Omega$$

c) The parameters referred to in per-unit are:

$$R_{eq-pu} = \frac{P_{sc}}{S_{Base}} = \frac{600W}{50kVA} = 0.012\,pu$$

or

$$Z_{2-Base} = \frac{(208V)^2}{50kVA} = 0.8653\Omega, \quad R_{eq-pu} = \frac{0.01036\Omega}{0.8653\Omega} = 0.012\,pu$$

$$Z_{eq-pu} = \frac{V_{sc-pu}}{I_{sc-pu}} = \frac{(370V/7200V)}{1.0\,pu} = 0.0513\,pu$$

$$X_{eq-pu} = \sqrt{(0.0513\,pu)^2 - (0.012\,pu)^2} = 0.050\,pu$$

$$R_{c-pu} = \frac{(V_{oc-pu})^2}{P_{oc-pu}} = \frac{(1.0\,pu)^2}{(500W/50kVA)} = 100\,pu$$

$$X_{m-pu} = \frac{V_{oc-pu}}{I_{oc-pu}\sin\theta_{oc}} = \frac{1.0\,pu}{(8.0A/138.8A)\cdot(0.985)} = 17.6\,pu$$

d) Regulation:

$$V_1 = 1.0\,pu\angle 0° + (1.0\,pu\angle -36.9°)(0.0513\,pu\angle 76.5°)$$
$$= 1.0395 + j0.0327 = 1.04\,pu\angle 1.8°$$

$$V.R. = \frac{1.04 - 1.0}{1.0} = 0.04 = 4.0\%$$

e) Full load efficiency is:

$$\eta_{FL} = \frac{\cos\phi}{\cos\phi + R_{eq-pu} + P_{c-pu}} = \frac{0.8}{0.8 + 0.012 + {}^{500W}\!/_{50kVA}} = 0.973 = 93.7\%$$

25% loading efficiency is:

$$\eta_{25\%} = \frac{0.25(0.8)}{0.25(0.8) + (0.25)^2(0.012) + 0.01} = 0.95 = 95\%$$