CHAPTER 12

12.1 to 12.4 $K_o = (1 - \sin\phi')(\text{OCR})^{\sin\phi'}$

Problem	ϕ' (deg)	K_o	$P_o = ½ K_o \gamma H^2$	$\bar{z} = \frac{H}{3}$
12.1	35	0.634	(½)(0.634)(18.1)(5)² = **143.44 kN / m**	**1.67 m**
12.2	30	0.5	(½)(0.5)(90)(16.5)² = **6125.6 lb / ft**	**5.5 ft**
12.3	38	0.675	(½)(0.675)(17)(5)² = **143.44 kN / m**	**1.67 m**
12.4	40	0.463	(½)(0.463)(115)(18)² = **8625.7 lb / ft**	**6 ft**

12.5 to 12.8 $K_a = \tan^2\left(45 - \frac{\phi'}{2}\right)$

Problem	ϕ' (deg)	K_a	$\sigma'_{a(z=H)} = K_a \gamma H$	$P_a = ½ K_a \gamma H^2$	$\bar{z} = \frac{H}{3}$
12.5	32	0.307	(0.307)(110)(10) = **337.7 lb / ft²**	(½)(0.307)(110)(10)² = **1688.5 lb / ft**	**3.33 ft**
12.6	28	0.361	(0.361)(98)(12) = **424.5 lb / ft²**	(½)(0.361)(98)(12)² = **2547 lb / ft**	**4 ft**
12.7	36	0.26	(0.26)(17.6)(3) = **13.73 kN / m²**	(½)(0.26)(17.6)(3)² = **20.59 kN / m**	**1 m**
12.8	40	0.217	(0.217)(18.2)(6) = **23.7 kN / m²**	(½)(0.217)(18.2)(6)² = **71.09 kN / m**	**2 m**

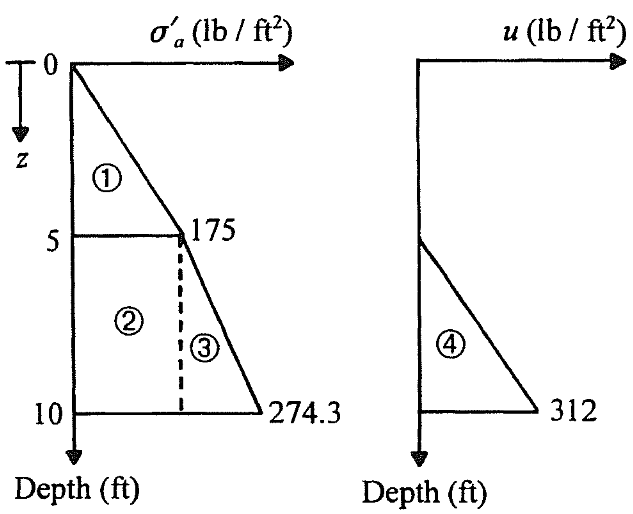
Note: 1. Pressure distribution is similar to that shown in Figure 12.13a; that is, $\sigma'_a = 0$ at $z = 0$ and $\sigma'_a = K_a\gamma H$ at $z = H$.
2. $\bar{z}$ is the distance measured from the bottom of the wall.

12.9 to 12.12 $K_p = \tan^2\left(45 + \frac{\phi'}{2}\right)$

Problem	ϕ' (deg)	K_p	$\sigma'_{p(z=H)} = K_p \gamma H$	$P_p = ½ K_p \gamma H^2$	$\bar{z} = \frac{H}{3}$
12.9	34	3.537	(3.537)(110)(10) = **3890.7 lb / ft²**	(½)(3.537)(110)(10)² = **19,454 lb / ft**	**3.33 ft**
12.10	36	3.852	(3.852)(105)(12) = **4853.5 lb / ft²**	(½)(3.852)(105)(12)² = **29,212 lb / ft**	**4 ft**
12.11	31	3.124	(3.124)(14.4)(5) = **224.9 kN / m²**	(½)(3.124)(14.4)(5)² = **562.3 kN / m**	**1.67 m**
12.12	28	2.77	(2.77)(13.5)(4) = **149.6 kN / m²**	(½)(2.77)(13.5)(4)² = **299.2 kN / m**	**1.33 m**

Note: 1. $\sigma'_{p(z=0)} = 0$; triangular pressure distribution
2. $\bar{z}$ is the distance measured from the bottom of the wall

12.13 $K_a = \tan^2\left(45 - \frac{\phi'}{2}\right) = \tan^2\left(45 - \frac{30}{2}\right) = \frac{1}{3}$. Refer to the figure.



$z = 0$ ft: $\sigma'_a = \sigma'_o K_a = 0; \quad u = 0$

$z = 5$ ft: $\sigma'_a = (105)(5)(⅓) = 175 \text{ lb / ft}^2; \quad u = 0$

$z = 10$ ft: $\sigma'_a = [(105)(5) + (122 - 62.4)(5)](⅓) = 274.3 \text{ lb / ft}^2$

$u = (62.4)(5) = 312 \text{ lb / ft}^2$

Area No.	Area
1	(½)(5)(175) = 437.5
2	(175)(5) = 875
3	(½)(5)(274.3 − 175) = 248.3
4	(½)(5)(312) = 780

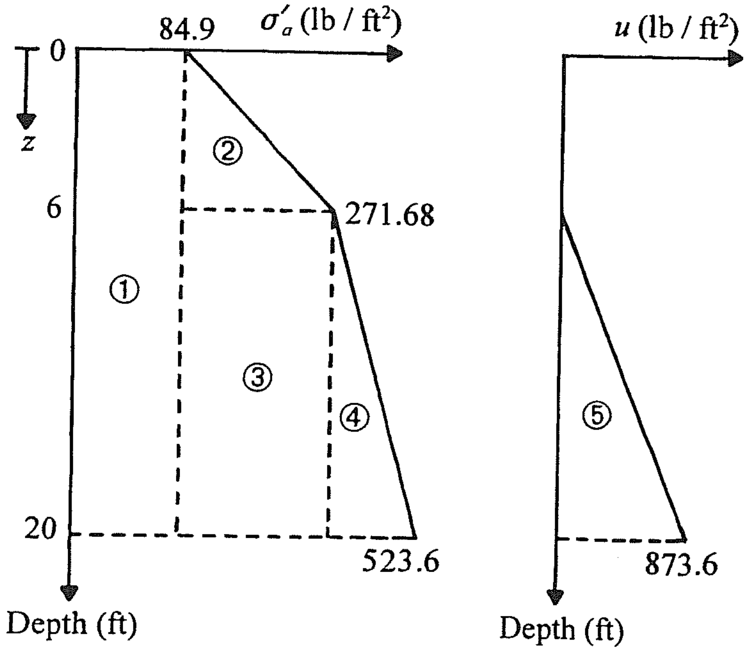
$P_a = \sum$**2,340.8 lb / ft**

Resultant: Taking the moment about the bottom of the wall,

$$\bar{z} = \frac{\left[(437.5)\left(5+\frac{5}{3}\right)+(875)\left(\frac{5}{2}\right)+(248.3)\left(\frac{5}{3}\right)+(780)\left(\frac{5}{3}\right)\right]}{2340.8}$$

$$= \frac{2916.7+2187.5+413.8+1300}{2340.8} = \mathbf{2.91\ ft}$$

12.14 $K_a = \tan^2\left(45-\frac{34}{2}\right) = 0.283$. Refer to the figure.



$z = 0$ ft: $\quad \sigma'_a = \sigma'_o K_a = (300)(0.283) = 84.9\ \text{lb / ft}^2;\ \ u = 0$

$z = 6$ ft: $\quad \sigma'_a = [300 + (6)(110)](0.283) = 271.68\ \text{lb / ft}^2;\ \ u = 0$

$z = 20$ ft: $\sigma'_a = [300 + (6)(110) + (126 - 62.4)(14)](0.283) = 523.66 \text{ lb / ft}^2$

$u = (62.4)(14) = 873.6 \text{ lb / ft}^2$

Area No.	Area
1	$(84.9)(20) = 1{,}698$
2	$(½)(6)(271.68 - 84.9) = 560.34$
3	$(14)(271.68 - 84.9) = 2{,}614.92$
4	$(½)(14)(523.6 - 271.68) = 1{,}763.44$
5	$(½)(14)(873.6) = 6{,}115.2$

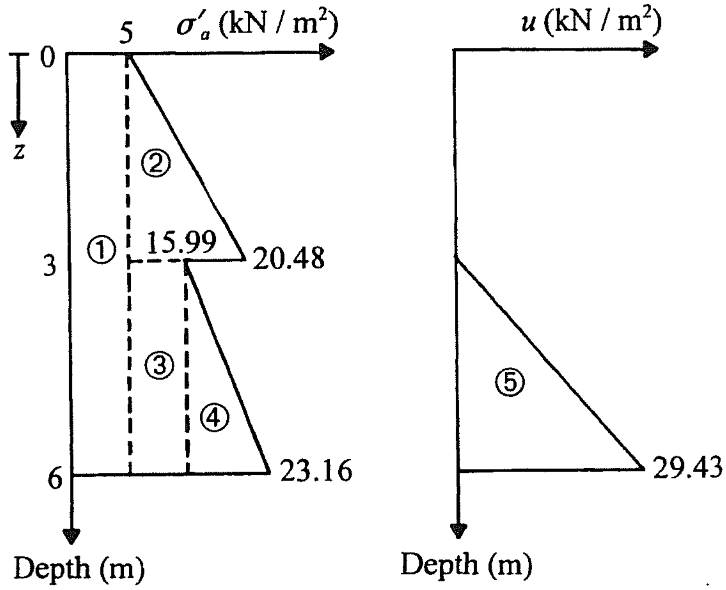
$P_a = \sum$ **12,751.9 lb / ft**

Location of resultant: Taking the moment about the bottom of the wall,

$$\bar{z} = \frac{\left[\begin{array}{l}(1{,}698)\left(\frac{20}{2}\right) + (560.34)\left(14 + \frac{6}{3}\right) + (2{,}614.92)\left(\frac{14}{2}\right) \\ +(1{,}763.44)\left(\frac{14}{3}\right) + (6{,}115.2)\left(\frac{14}{3}\right)\end{array}\right]}{12{,}751.9}$$

$$= \frac{16{,}980 + 8{,}965.44 + 18{,}304.44 + 8{,}229.4 + 28{,}537.6}{12{,}751.9} = \mathbf{6.35 \text{ ft}}$$

12.15 $K_{a(1)} = \tan^2\left(45 - \frac{30}{2}\right) = 0.333$; $K_{a(2)} = \tan^2\left(45 - \frac{36}{2}\right) = 0.26$. Refer to the figure.



$z = 0$ m: $\sigma'_a = \sigma'_o K_{a(1)} = (15)(0.333) = 5 \text{ kN/m}^2$; $u = 0$

$z = 3$m: $\sigma'_a = \sigma'_o K_{a(1)} = [(15.5)(3) + 15](0.333) = 20.48 \text{ kN/m}^2$

$\sigma'_a = \sigma'_o K_{a(2)} = [(15.5)(3) + 15](0.26) = 15.99 \text{ kN/m}^2$

$u = 0$

$z = 6$ m: $\sigma'_a = \sigma'_o K_{a(2)} = [15 + (15.5)(3) + (19 - 9.81)(3)](0.26) = 23.16 \text{ kN/m}^2$

$u = (9.81)(3) = 29.43 \text{ kN/m}^2$

Area No.	Area
1	$(6)(5) = 30$
2	$(½)(3)(20.48 - 5) = 23.22$
3	$(3)(15.99 - 5) = 32.97$
4	$(½)(3)(23.16 - 15.99) = 10.76$
5	$(½)(3)(29.43) = 44.15$
	$P_a = \sum$ **141.1 kN/m**

Location of resultant: Taking the moment about the bottom

$$\bar{z} = \frac{(30)\left(\frac{6}{2}\right) + (23.22)\left(3 + \frac{3}{3}\right) + (32.97)\left(\frac{3}{2}\right) + (10.76)\left(\frac{3}{3}\right) + (44.15)\left(\frac{3}{3}\right)}{141.1}$$

$$= \frac{90 + 92.88 + 49.46 + 10.76 + 44.15}{141.1} = \frac{287.25}{141.1} = \mathbf{2.04\ m}$$

12.16 a. Equation (12.23):

$$\sigma'_a = \frac{\gamma z \cos\alpha \sqrt{1 + \sin^2\phi' - 2\sin\phi' \cos\psi_a}}{\cos\alpha + \sqrt{\sin^2\phi' - \sin^2\alpha}}$$

$$\psi_a = \sin^{-1}\left(\frac{\sin\alpha}{\sin\phi'}\right) - \alpha + 2\theta = \sin^{-1}\left(\frac{\sin 10}{\sin 30}\right) - 10 + (2)(5) = 20.32°$$

$$\sigma'_a = \frac{(15)(4)(\cos 10)\sqrt{1 + \sin^2 30 - (2)(\sin 30)(\cos 20.32)}}{\cos 10 + \sqrt{\sin^2 30 - \sin^2 10}} = \mathbf{22.7\ kN/m^2}$$

Equation (12.25):

$$\beta = \tan^{-1}\left(\frac{\sin\phi'\sin\psi_a}{1-\sin\phi'\cos\psi_a}\right) = \tan^{-1}\left[\frac{(\sin 30)(\sin 20.32)}{1-(\sin 30)(\cos 20.32)}\right] = \mathbf{18.1°}$$

b. Equation (12.27):

$$K_{a(R)} = \frac{\cos(\alpha-\theta)\sqrt{1+\sin^2\phi' - 2\sin\phi'\cos\psi_a}}{\cos^2\theta\left(\cos\alpha+\sqrt{\sin^2\phi'-\sin^2\alpha}\right)}$$

$$= \frac{\cos(10-5)\sqrt{1+\sin^2 30-(2)(\sin 30)(\cos 20.32)}}{\cos^2 10\left(\cos 10+\sqrt{\sin^2 30-\sin^2 10}\right)} = 0.394$$

$$P_a = \frac{1}{2}\gamma H^2 K_{a(R)} = \frac{1}{2}(15)(4)^2(0.394) = \mathbf{47.28\ kN/m}$$

Location and direction of resultant: – **At a distance of $H/3 = 4/3 = 1.33$ m above the bottom of the wall inclined at an angle $\beta = 18.1°$ to the normal drawn to the back face of the wall**

12.17 This is a Rankine case since $\delta' = 0$. $P_p = \frac{1}{2}\gamma H^2 K_{p(R)}$

Equation (12.33): $$K_{p(R)} = \frac{\cos(\alpha-\theta)\sqrt{1+\sin^2\phi' + 2\sin\phi'\cos\psi_p}}{\cos^2\theta\left(\cos\alpha-\sqrt{\sin^2\phi'-\sin^2\alpha}\right)}$$

$\alpha = 0$; $\theta = 10°$; $\phi' = 36°$

$$\psi_p = \sin^{-1}\left(\frac{\sin\alpha}{\sin\phi'}\right)+\alpha-2\theta = \sin^{-1}\left(\frac{\sin 0}{\sin 36}\right)+0-(2)(10) = -20°$$

$$K_{p(R)} = \frac{\cos(0-10)\sqrt{1+\sin^2 36+(2)(\sin 36)\cos(-20)}}{\cos^2(10)\left(\cos 0-\sqrt{\sin^2 36-\sin^2 0}\right)} = 3.855$$

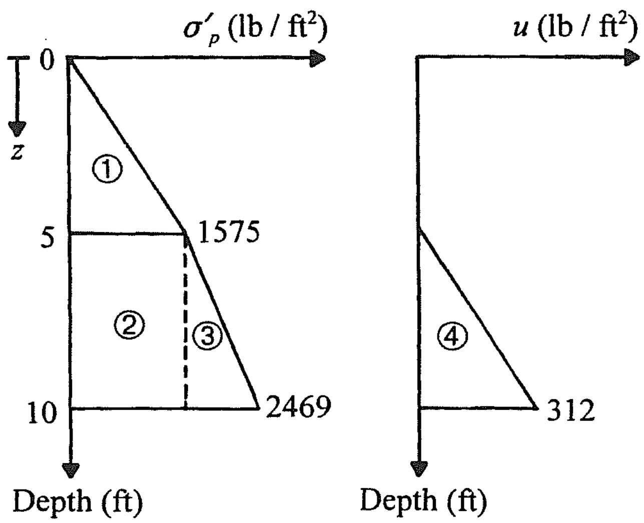
$$P_p = \frac{1}{2}(16.5)(3)^2(3.855) = \mathbf{286.2\ kN/m}$$

Equation (12.32):

$$\beta = \tan^{-1}\left(\frac{\sin\phi'\sin\psi_p}{1+\sin\phi'\cos\psi_p}\right) = \tan^{-1}\left\{\frac{\sin 36\sin(-20)}{1+(\sin 36)[\cos(-20)]}\right\} = \textbf{-7.34°}$$

P_p acts at a distance of $H/3 = 3/3 = 1$ m from the bottom of the wall inclined at an angle $\beta = -7.34°$ to the normal drawn to the back face of the wall.

12.18 $K_p = \tan^2\left(45 + \frac{30}{2}\right) = 3$. Refer to the figure.



$z = 0$ ft: $\sigma_p' = 0$; $u = 0$

$z = 5$ ft: $\sigma_p' = \gamma_1 z K_p = (105)(5)(3) = 1575$ lb / ft²; $u = 0$

$z = 10$ ft: $\sigma_p' = [(105)(5) + (122 - 62.4)(5)](3) = 2469$ lb / ft²

$u = (62.4)(5) = 312$ lb / ft²

Area No.	Area
1	(½)(5)(1575) = 3,937.5
2	(5)(1575) = 7,875
3	(½)(5)(2469 − 1575) = 2,235
4	(½)(5)(312) = 780

$P_p = \sum$ **14,828 lb / ft**

Location of the resultant: Taking the moment about the bottom of the wall,

$$\bar{z} = \frac{(3937.5)\left(5+\frac{5}{3}\right)+(7875)\left(\frac{5}{2}\right)+(2235)\left(\frac{5}{3}\right)+(780)\left(\frac{5}{3}\right)}{14{,}828} = \mathbf{3.44\ ft}$$

12.19 a. $H = 4.5$ m; $c_u = 19.3$ kN / m²; $\gamma = 19.6$ kN / m³; $\phi = 0$

$$K_a = \tan^2\left(45-\frac{\phi}{2}\right) = 1; \quad \sigma_a = \gamma z - 2c_u$$

At the top ($z = 0$):

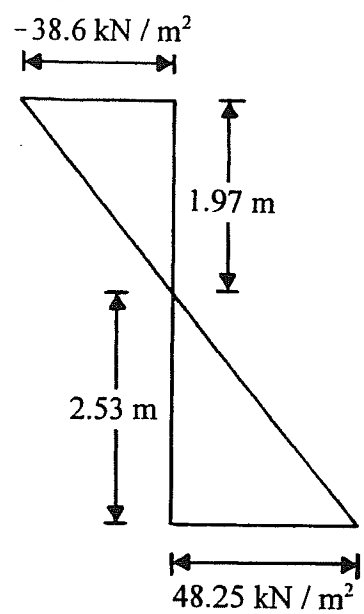
$$\sigma_a = -2c_u = (-2)(19.3) = -38.6 \text{ kN / m}^2$$

At the bottom ($z = 4.5$ m):

$$\sigma_a = (19.3)(4.5) - (2)(19.3)$$

$$= 86.85 - 38.6 = 48.25 \text{ kN / m}^2$$

The pressure diagram is shown.



b. Equation (12.49):

$$z_o = \frac{2c_u}{\gamma} = \frac{(2)(19.3)}{19.6} = \mathbf{1.97\ m}$$

c. Equation (12.51):

$$P_a = \frac{1}{2}\gamma H^2 - 2c_u H = \left(\frac{1}{2}\right)(19.6)(4.5)^2 - (2)(19.3)(4.5) = \mathbf{24.75\ kN / m}$$

d. Equation (12.53):

$$P_a = \frac{1}{2}\gamma H^2 - 2c_u H + \frac{2c_u^2}{\gamma} = \left(\frac{1}{2}\right)(19.6)(4.5)^2 - (2)(19.3)(4.5) + \frac{(2)(19.3)^2}{19.6}$$

$$= \mathbf{62.76\ kN / m}$$

Resultant measured from the bottom:

$$\frac{H-z_o}{3}=\frac{4.5-1.97}{3}=\mathbf{0.84\ m}$$

12.20 a. $\sigma_a=\sigma_o K_a-2c_u\sqrt{K_a}$

$\sigma_o=\gamma z+q;\quad K_a=1$

At $z=0$:

$\sigma_o=8\ \text{kN/m}^2$

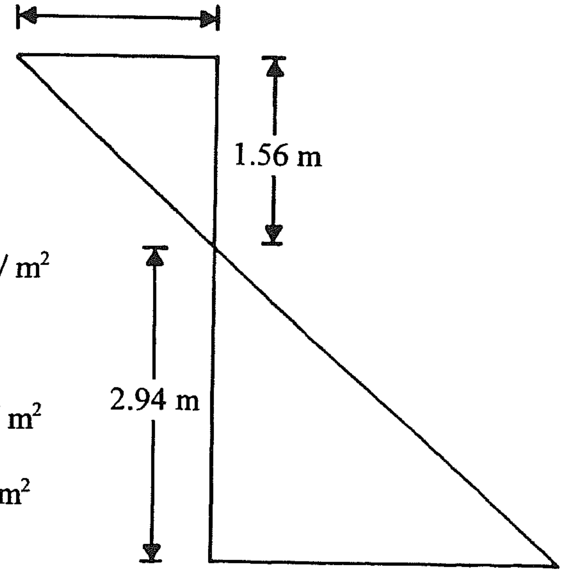
$\sigma_a=8-(2)(19.3)=-30.6\ \text{kN/m}^2$

At $z=4.5$ m:

$\sigma_o=(19.6)(4.5)+8=96.2\ \text{kN/m}^2$

$\sigma_a=\sigma_o-(2)(19.3)=57.6\ \text{kN/m}^2$

The pressure diagram is shown.



b. $\sigma_a=0.\ (\gamma z+q)-2c_u=0.$

$$z_o=\frac{2c_u-q}{\gamma}=\frac{38.6-8}{19.6}=\mathbf{1.56\ m}$$

c. Referring to the diagram in Part a:

$$P_a=\left(\frac{1}{2}\right)(2.94)(57.6)-\left(\frac{1}{2}\right)(30.6)(1.56)=\mathbf{60.8\ kN/m}$$

d. $P_a=\left(\frac{1}{2}\right)(2.94)(57.6)=\mathbf{84.67\ kN/m}$

Location of the resultant from the bottom of the wall $=\frac{2.94}{3}=\mathbf{0.98\ m}$

12.21 $K_a = \tan^2\left(45 - \frac{\phi'}{2}\right) = \tan^2\left(45 - \frac{16}{2}\right) = 0.568; \quad \sqrt{K_a} = 0.754$. Equation (12.52):

$$P_a = \frac{1}{2}K_a\gamma H^2 - 2\sqrt{K_a}c'H + \frac{2c'^2}{\gamma}$$

$$= \frac{1}{2}(0.568)(19)(5)^2 - (2)(0.754)(26)(5) + \frac{(2)(26)^2}{19} = \mathbf{10.02\ kN/m}$$

12.22 Equation (12.63):

$$z_o = \frac{2c'}{\gamma}\sqrt{\frac{1+\sin\phi'}{1-\sin\phi'}} = \frac{(2)(88)}{110}\sqrt{\frac{1+\sin 25}{1-\sin 25}} = 2.51 \text{ ft}$$

At $z = 0$ ft: $\sigma'_a = 0$

At $z = 15$ ft: $\sigma'_a = \gamma z K'_{a(R)} \cos\alpha$

$$\frac{c'}{\gamma z} = \frac{88}{(110)(15)} = 0.053$$

For $\alpha = 10°$, $\phi' = 25°$, and $\frac{c'}{\gamma z} = 0.053$, the value of $K'_{a(R)} \approx 0.366$

$$\sigma'_a = (110)(15)(0.366)(\cos 10) = 594.7 \text{ lb/ft}^2$$

$$P_a = \frac{1}{2}(15 - 2.51)(594.7) = \mathbf{3714\ lb/ft}$$

12.23 Use Equations (12.68) and (12.69).

$\alpha = 0$; $\theta = 10°$; $\phi' = 36°$; $\gamma = 18$ kN/m^3; $H = 5$ m

Part	δ' (deg)	K_a [Equation (12.69)]	$P_a = \frac{1}{2}K_a\gamma H^2$ [Equation (12.68)]
1	18	0.3118	70.15 kN/m
2	24	0.3137	70.58 kN/m

P_a is located at a vertical distance of 5/3 = 1.67 m above the bottom of the wall and is inclined at an angle δ' to the normal drawn to the back face of the wall.

12.24 a. $\phi' = 38°$; $\psi = 90 - \theta - \delta' = 90 - 5 - 20 = 65°$

$$\text{Weight of wedge } ABC = \frac{1}{2}(11.6)(7.1)(\underbrace{128}_{\gamma}) = 5721 \text{ lb/ft} = 5.271 \text{ kip/ft}$$

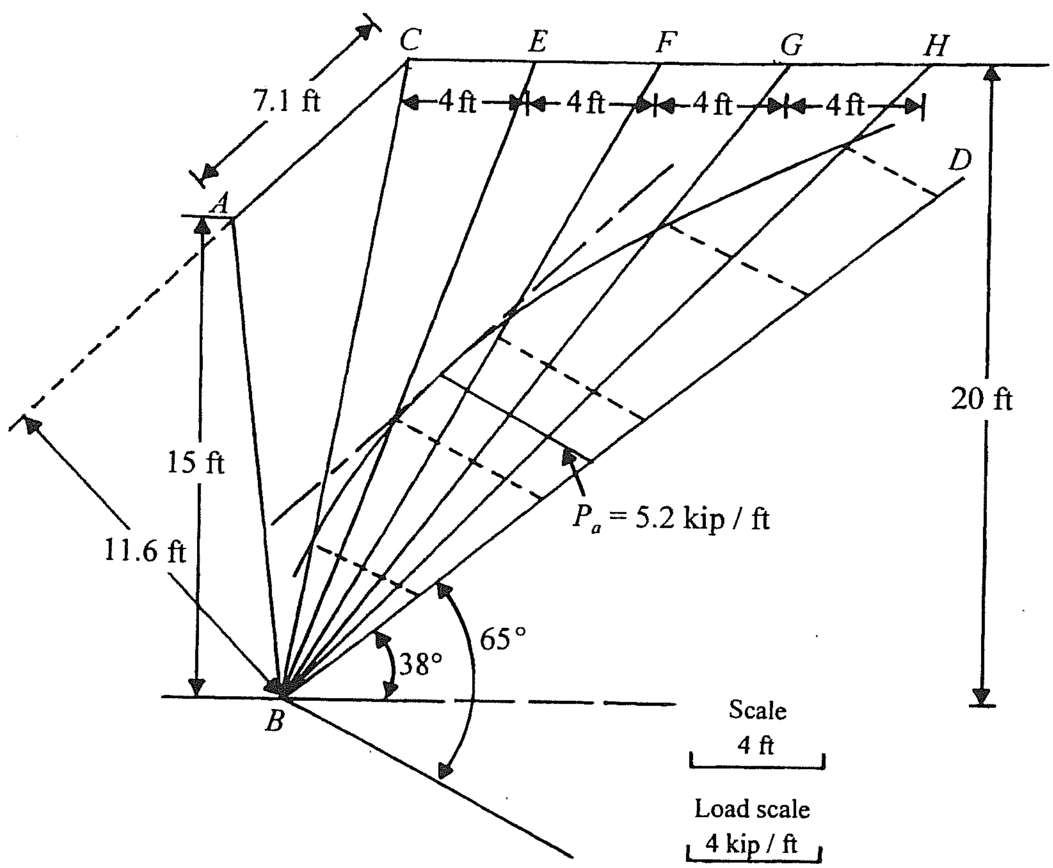
The weight of each of the wedges *CBE, EBF, FBG, GBH* =

$$\left(\frac{1}{2}\right)(20)(4)(128) = 5120 \text{ lb/ft} = 5.12 \text{ kip/ft}$$

Wedge	Weight (kip / ft)
ABC	5.271
ABE	5.271 + 5.12 = 10.391
ABF	10.391 + 5.12 = 15.511
ABG	15.511 + 5.12 = 20.631
ABH	20.631 + 5.12 = 25.751

The graphical construction is shown.

P_a = **5.2 kip / ft.**



b. $\phi' = 34°$; $\psi = 90 - 0 - 17 = 73°$

$$\text{Weight of wedge } ABC = \frac{1}{2}(16)(9)(116) = 8352 \text{ lb/ft} \approx 8.35 \text{ kip/ft}$$

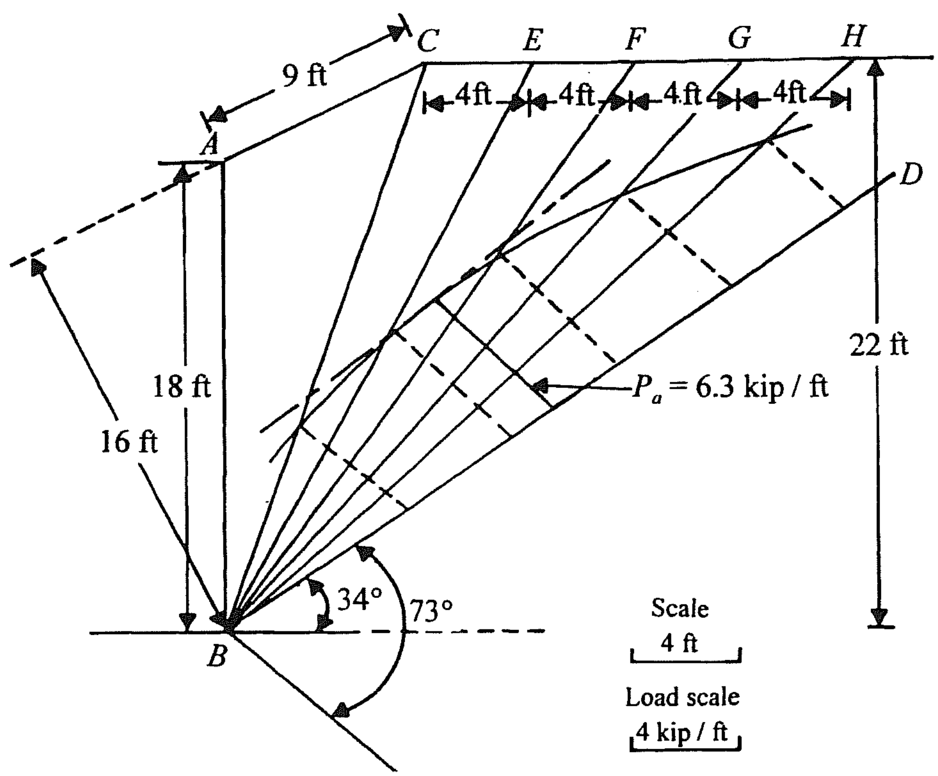
The weight of each of the wedges *CBE, EBF, FBG, GBH* =

$$\left(\frac{1}{2}\right)(4)(22)(116) = 5104 \text{ lb/ft} \approx 5.10 \text{ kip/ft}$$

Wedge	Weight (kip / ft)
ABC	8.35
ABE	8.35 + 5.10 = 13.45
ABF	13.45 + 5.10 = 18.55
ABG	18.55 + 5.10 = 23.65
ABH	23.65 + 5.10 = 28.75

The graphical construction is shown.

P_a = **6.3 kip / ft.**



c. $\gamma = \dfrac{(1680)(9.81)}{1000} = 16.48\ \text{kN/m}^3;\ \phi' = 30°;\ \psi = 90 - 10 - 30 = 50°$

$$\text{Weight of wedge } ABC = \frac{1}{2}(5.25)(2.5)(16.48) = 108.15\ \text{kN/m}$$

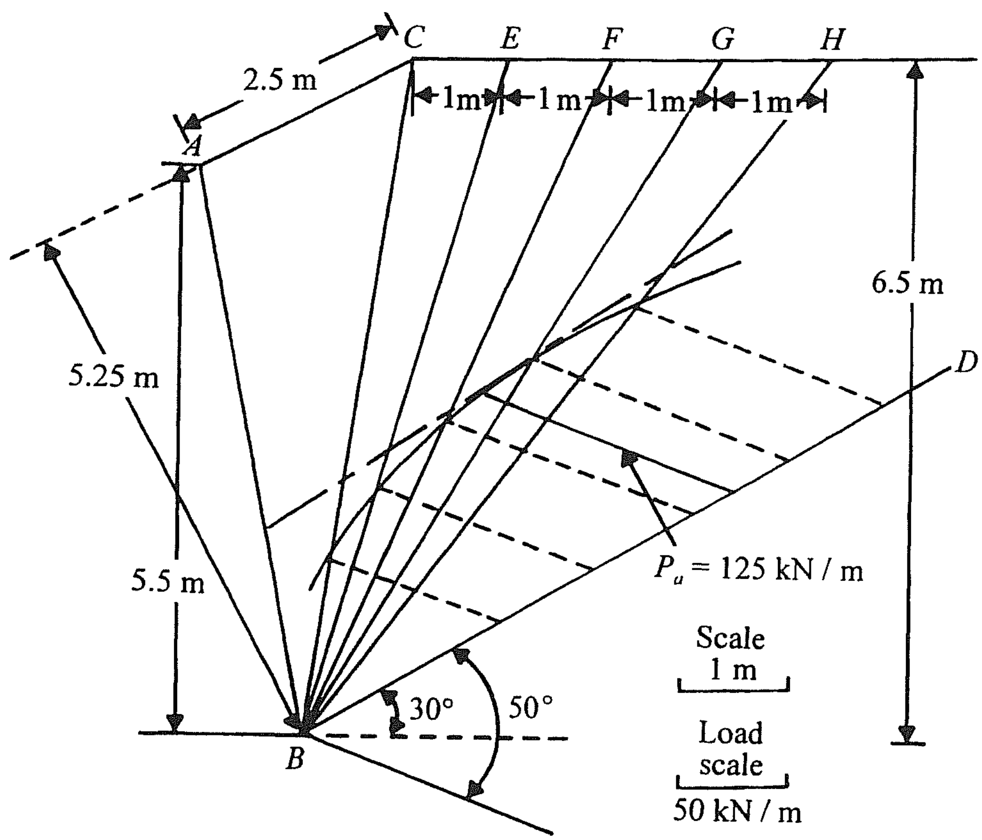
The weight of each of the wedges *CBE, EBF, FBG, GBH* =

$$\left(\frac{1}{2}\right)(1)(6.5)(16.48) = 53.46\ \text{kN/m}$$

Wedge	Weight (kN / m)
ABC	108.15
ABE	108.15 + 53.56 = 161.71
ABF	161.71 + 53.56 = 215.27
ABG	215.27 + 53.56 = 268.83
ABH	268.83 + 53.56 = 322.39

The graphical construction is shown.

P_a = **125 kN / m.**



12.25 Equation (12.74): $P_{ae} = \frac{1}{2}\gamma H^2 (1-k_v)K_a''$

$k_v = 0;\ \theta = 0;\ \alpha = 0;\ \phi' = 35°; \frac{\delta}{\phi'} = \frac{2}{3};\ k_h = 0.3$. From Table 12.9, $K_a'' = 0.486$

$$P_{ae} = \frac{1}{2}(15)(6)^2(1 - 0)(0.486) = \mathbf{131.2\ kN/m}$$

For $\phi' = 35°$, $\frac{\delta'}{\phi'} = \frac{2}{3}$. From Table 12.6, $K_a = 0.2444$.

$$P_a = \frac{1}{2}\gamma H^2 K_a = \frac{1}{2}(15)(6)^2(0.2444) = 66\ \text{kN/m}$$

Equation (12.83):

$$\bar{z} = \frac{P_a\left(\frac{H}{3}\right) + \Delta P_{ae}(0.6H)}{P_{ae}} = \frac{(66)\left(\frac{6}{3}\right) + (131.2 - 66)(0.6\times 6)}{131.2} = \mathbf{2.8\ m}$$

12.26 Equation (12.84): $P_{ae} = \gamma(H - z_o)^2 N'_{a\gamma} - c'(H - z_o)^2 N'_{ac}$

Given: $z_o = 0;\ \theta = 10°;\ \phi' = 15°;\ k_h = 0.15$

$N'_{ac} = N_{ac} = 1.75$ (Figure 12.33); $N_{a\gamma} = 0.3$ (Figure 12.35); $\lambda = 1.3$ (Figure 12.36);

$N'_{a\gamma} = \lambda N_{a\gamma}$. So,

$$P_{ae} = (19)(6 - 0)^2(1.3 \times 0.3) - (20)(6 - 0)(1.75) = \mathbf{56.76\ kN/m}$$

12.27 $z_o = 0;\ n = 0;\ \theta = 5°;\ \phi' = 20°;\ k_h = 0.25;\ N'_{ac} \approx 1.65;\ N_{a\gamma} \approx 0.25;\ \lambda \approx 1.65;\ N'_{a\gamma} = \lambda N_{a\gamma}$

Equation (12.84):

$$P_{ae} = \gamma(H - z_o)^2 N'_{a\gamma} - c'(H - z_o)^2 N'_{ac}$$

$$= (100)(10 - 0)^2(1.65 \times 0.25) - (200)(10 - 0)(1.65)$$

$$= 4125 - 3300 = \mathbf{825\ lb/ft}$$