

CHAPTER 11

- 11.1 a. Normal stress, $\sigma' = 192 \text{ kN / m}^2$; shear stress, $\tau_f = 120 \text{ kN / m}^2$

$$\phi' = \tan^{-1}\left(\frac{\tau_f}{\sigma'}\right) = \tan^{-1}\left(\frac{120}{192}\right) = 32^\circ$$

- b. $\tau_f = \sigma' \tan \phi' = 200 \tan 32 = 124.97 \text{ kN / m}^2$

$$\text{Shear force, } S = (\tau_f) \left(\frac{50 \times 50}{1000 \times 1000} \right) = (124.97)(0.05)^2 = 0.312 \text{ kN} = 312 \text{ N}$$

- 11.2 Shear force, $S = (2 \times 2 \text{ in.}^2)(\sigma' \tan \phi') = (4)(15)(\tan 41) = 52.16 \text{ lb}$

- 11.3 Area of specimen $= \left(\frac{\pi}{4}\right)(0.05)^2 = 0.0019634 \text{ m}^2 = A$

| Test No. | Normal force, N (N) | $\sigma' = \frac{N}{A}$ (kN / m ²) | Shear force, S (N) | $\tau_f = \frac{S}{A}$ (kN / m ²) | $\phi' = \tan^{-1}\left(\frac{\tau_f}{\sigma'}\right)$ |
|----------|-----------------------|--|----------------------|---|--|
| 1 | 250 | 127.3 | 139 | 70.8 | 29.08 |
| 2 | 400 | 203.7 | 222 | 113.1 | 29.04 |
| 3 | 500 | 254.7 | 279 | 142.1 | 29.16 |
| 4 | 550 | 280.1 | 308 | 156.9 | 20.26 |

A graph of τ_f versus σ' will yield $\phi' \approx 29.1^\circ$.

- 11.4 Area, $A = 0.0019634 \text{ m}^2$

| Test No. | Normal force, N (N) | $\sigma' = \frac{N}{A}$ (kN / m ²) | Shear force, S (N) | $\tau_f = \frac{S}{A}$ (kN / m ²) | $\phi' = \tan^{-1}\left(\frac{\tau_f}{\sigma'}\right)$ |
|----------|-----------------------|--|----------------------|---|--|
| 1 | 200 | 101.9 | 82 | 41.8 | 22.3 |
| 2 | 300 | 152.8 | 120 | 61.1 | 21.8 |
| 3 | 400 | 203.7 | 160 | 81.5 | 21.8 |
| 4 | 550 | 280.1 | 220 | 112.1 | 21.8 |

A graph of τ_f versus σ' will yield $\phi' \approx 21.9^\circ$.

11.5 Equation (11.8). With $c' = 0$

$$\sigma'_1 = \sigma'_3 \tan^2 \left(45 + \frac{\phi'}{2} \right) = 105 \tan^2 \left(45 + \frac{36}{2} \right) = 404.4 \text{ kN / m}^2$$

$$\Delta\sigma_{d(\theta)} = 404.4 - 105 = 299.4 \text{ kN / m}^2$$

11.6 a. Equation (11.4):

$$\theta = 45 + \frac{\phi'}{2} = 45 + \frac{36}{2} = 63^\circ$$

b. From Equations (9.8) and (9.9):

$$\begin{aligned} \sigma' &= \frac{\sigma'_1 + \sigma'_3}{2} + \frac{\sigma'_1 - \sigma'_3}{2} \cos 2\theta = \frac{404.4 + 105}{2} + \frac{404.4 - 105}{2} \cos(2 \times 25) \\ &= 350.8 \text{ kN / m}^2 \end{aligned}$$

$$\tau = \frac{\sigma'_1 - \sigma'_3}{2} \sin 2\theta = \frac{404.4 - 105}{2} \sin(2 \times 25) = 114.68 \text{ kN / m}^2$$

$$\text{For failure, } \tau_f = \sigma' \tan \phi' = 350.8 \tan 36 = 254.9 \text{ kN / m}^2$$

Since the developed shear stress τ is 114.68 kN / m², which is less than 254.9 kN / m²,

the specimen did not fail.

11.7 $\phi' = 25 + 0.18D_r = 25 + (0.18)(60) = 35.8^\circ$

$$\sigma'_1 = \sigma'_3 \tan^2 \left(45 + \frac{\phi'}{2} \right) = 18 \tan^2 \left(45 + \frac{35.8}{2} \right) = 68.7 \text{ lb / in.}^2$$

11.8 $\sigma'_1 = \sigma'_3 + \Delta\sigma_{d(\theta)} = 140 + 264 = 404 \text{ kN / m}^2$

$$\sigma'_1 = \sigma'_3 \tan^2 \left(45 + \frac{\phi'}{2} \right)$$

$$\phi' = 2 \left[\tan^{-1} \left(\frac{\sigma'_1}{\sigma'_3} \right)^{0.5} - 45 \right] = 2 \left[\tan^{-1} \left(\frac{404}{140} \right)^{0.5} - 45 \right] = 29.03^\circ$$

$$11.9 \quad \sigma'_1 = \sigma'_3 + \Delta \sigma_{d(f)} = \sigma'_3 \tan^2 \left(45 + \frac{\phi'}{2} \right)$$

$$\sigma'_3 = \frac{\Delta \sigma_{d(f)}}{\tan^2 \left(45 + \frac{\phi'}{2} \right) - 1} = \frac{23}{\tan^2 \left(45 + \frac{26}{2} \right) - 1} = 14.73 \text{ lb / in.}^2$$

$$11.10 \quad \sigma'_3 = \frac{\Delta \sigma_{d(f)}}{\tan^2 \left(45 + \frac{\phi'}{2} \right) - 1} = \frac{180}{\tan^2 \left(45 + \frac{25}{2} \right) - 1} = 122.96 \text{ kN / m}^2$$

$$11.11 \text{ a. } \sigma'_1 = \sigma'_3 \tan^2 \left(45 + \frac{\phi'}{2} \right); \quad (300 + 350) = 300 \tan^2 \left(45 + \frac{\phi'}{2} \right)$$

$$\phi' = 21.6^\circ$$

$$\text{b. } \theta = 45 + \frac{\phi'}{2} = 45 + \frac{21.6}{2} = 55.8^\circ$$

c. From Equations (9.8) and (9.9):

$$\sigma' = \frac{\sigma'_1 + \sigma'_3}{2} + \frac{\sigma'_1 - \sigma'_3}{2} \cos 2\theta = \frac{650 + 300}{2} + \frac{650 - 300}{2} \cos(2 \times 55.8)$$

$$= 410.6 \text{ kN / m}^2$$

$$\tau = \frac{\sigma'_1 - \sigma'_3}{2} \sin 2\theta = \frac{650 - 300}{2} \sin(2 \times 55.8) = 162.7 \text{ kN / m}^2$$

$$11.12 \quad \sigma'_1 = \sigma'_3 \tan^2 \left(45 + \frac{\phi'}{2} \right) + 2c' \tan \left(45 + \frac{\phi'}{2} \right)$$

$$\text{Specimen I: } (15 + 31.4) = 46.4 = 15 \tan^2 \left(45 + \frac{\phi'}{2} \right) + 2c' \tan \left(45 + \frac{\phi'}{2} \right) \quad (\text{a})$$

$$\text{Specimen II: } (25 + 47) = 72 = 25 \tan^2 \left(45 + \frac{\phi'}{2} \right) + 2c' \tan \left(45 + \frac{\phi'}{2} \right) \quad (\text{b})$$

Equation (b) minus Equation (a):

$$72 - 46.4 = 10 \tan^2 \left(45 + \frac{\phi'}{2} \right); \quad \phi' = 26^\circ$$

From Equation (b):

$$c' = \frac{72 - 25 \tan^2 \left(45 + \frac{26}{2} \right)}{2 \tan \left(45 + \frac{26}{2} \right)} = 2.49 \text{ lb / in.}^2$$

$$11.13 \quad \sigma'_1 = 36 \tan^2 \left(45 + \frac{26}{2} \right) + (2)(2.49) \tan \left(45 + \frac{26}{2} \right) = 100.2 \text{ lb / in.}^2$$

$$11.14 \quad c' = 0$$

$$\sigma'_1 = \sigma'_3 + \Delta \sigma_{d(f)} = \sigma'_3 \tan^2 \left(45 + \frac{\phi'}{2} \right)$$

$$\sigma'_3 = \frac{\Delta \sigma_{d(f)}}{\tan^2 \left(45 + \frac{\phi'}{2} \right) - 1} = \frac{1.9}{\tan^2 \left(45 + \frac{38}{2} \right) - 1} = 0.59 \text{ ton / ft}^2$$

$$11.15 \quad \sigma_1 = \sigma_3 \tan^2 \left(45 + \frac{\phi}{2} \right); \quad \phi = 2 \left[\tan^{-1} \left(\frac{12 + 9.14}{12} \right)^{0.5} - 45 \right] = 16^\circ$$

$$\sigma'_1 = \sigma'_3 \tan^2 \left(45 + \frac{\phi'}{2} \right); \quad \phi' = 2 \left[\tan^{-1} \left(\frac{12 + 9.14 - 6.83}{12 - 6.83} \right)^{0.5} - 45 \right] = 28^\circ$$

$$11.16 \quad \phi = 2 \left[\tan^{-1} \left(\frac{140 + 125}{140} \right)^{0.5} - 45 \right] = 18^\circ$$

$$\phi' = 2 \left[\tan^{-1} \left(\frac{140 + 125 - 75}{140 - 75} \right)^{0.5} - 45 \right] = 29.4^\circ$$

$$11.17 \quad \text{a. } \sigma_1 = \sigma_3 \tan^2 \left(45 + \frac{\phi}{2} \right)$$

$$\phi = 2 \left[\tan^{-1} \left(\frac{105 + 97}{105} \right)^{0.5} - 45 \right] = 18.4^\circ$$

$$\text{b. } \sigma'_1 = \sigma'_3 \tan^2 \left(45 + \frac{\phi'}{2} \right)$$

$$\frac{105 + 97 - \Delta u_{d(f)}}{105 - \Delta u_{d(f)}} = \tan^2 \left(45 + \frac{28}{2} \right)$$

$$\Delta u_{d(f)} = \mathbf{50.2 \text{ kN / m}^2}$$

$$11.18 \quad \sigma'_1 = \sigma'_3 \tan^2 \left(45 + \frac{\phi'}{2} \right) = 105 \tan^2 \left(45 + \frac{28}{2} \right) = 290.8 \text{ kN / m}^2$$

$$\Delta \sigma_{d(f)} = 290.8 - 105 = \mathbf{185.8 \text{ kN / m}^2}$$

$$11.19 \quad \sigma_1 = \sigma_3 \tan^2 \left(45 + \frac{\phi}{2} \right) = 15 \tan^2 \left(45 + \frac{22}{2} \right) = 32.97 \text{ lb / in.}^2$$

$$\Delta \sigma_{d(f)} = 32.97 - 15 = \mathbf{17.97 \text{ lb / in.}^2}$$

$$\sigma'_1 = \sigma'_3 \tan^2 \left(45 + \frac{\phi'}{2} \right)$$

$$\frac{32.97 - \Delta \sigma_{d(f)}}{15 - \Delta \sigma_{d(f)}} = \tan^2 \left(45 + \frac{32}{2} \right)$$

$$\Delta u_{d(f)} = \mathbf{7.01 \text{ lb / in.}^2}$$

$$11.20 \quad q_u = \sigma_1 - \sigma_3 = 2800 - 1500 = \mathbf{1300 \text{ lb / ft}^2}$$

$$11.21 \quad \sigma'_1 = \sigma'_3 \tan^2 \left(45 + \frac{\phi'}{2} \right)$$

$$\frac{100 - \Delta \sigma_{d(f)}}{0 - \Delta \sigma_{d(f)}} = \tan^2 \left(45 + \frac{25}{2} \right)$$

$$\Delta u_{d(f)} = \mathbf{-68.5 \text{ kN / m}^2}$$

$$11.22 \quad \frac{140 - \Delta\sigma_{d(f)}}{0 - \Delta\sigma_{d(f)}} = \tan^2\left(45 + \frac{20}{2}\right)$$

$$\Delta u_{d(f)} = -134.6 \text{ kN / m}^2$$

11.23 a.

| Test No. | $\frac{\sigma'_1 + \sigma'_3}{2} = p'$ (lb / in. ²) | $\frac{\sigma'_1 - \sigma'_3}{2} = q'$ (lb / in. ²) |
|----------|--|--|
| 1 | 50 | 23 |
| 2 | 30 | 18 |

$$q' = m + p' \tan \alpha$$

$$23 = m + 50 \tan \alpha \quad (a)$$

$$18 = m + 30 \tan \alpha \quad (b)$$

$$m = 10.5 \text{ lb / in.}^2; \quad \alpha = 14^\circ$$

b. $\phi' = \sin^{-1}(\tan \alpha) = \sin^{-1}(\tan 14) = 14.44^\circ$

$$c' = \frac{m}{\cos \alpha} = \frac{10.5}{\cos(14)} = 10.82 \text{ lb / in.}^2$$

$$11.24 \quad \frac{c_{u(VST)}}{\sigma'_o} = 0.11 + 0.0037 PI$$

$$\sigma' = (3)(17) + (3)(19.5 - 9.81) = 80.07 \text{ kN / m}^2$$

$$c_{u(VST)} = [0.11 + (0.0037)(18)](80.07) = 14.14 \text{ kN / m}^2$$