CHAPTER 9

9.1 a. $$\left.\begin{matrix}\sigma_1\\ \sigma_3\end{matrix}\right\} = \frac{\sigma_y + \sigma_x}{2} \pm \sqrt{\left(\frac{\sigma_y - \sigma_x}{2}\right)^2 + \tau_{xy}^2}$$

$\sigma_x = 80$ kN / m^2; $\sigma_y = 120$ kN / m^2; $\tau_{xy} = +40$ kN / m^2; $\theta = 145°$

$$\left.\begin{matrix}\sigma_1\\ \sigma_3\end{matrix}\right\} = \frac{120+80}{2} \pm \sqrt{\left(\frac{120-80}{2}\right)^2 + (40)^2}$$

$\sigma_1 =$ **144.7 kN / m²**; $\sigma_3 =$ **55.3 kN / m²**

b. $$\sigma_n = \frac{\sigma_y + \sigma_x}{2} + \frac{\sigma_y - \sigma_x}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

$$= \frac{120+80}{2} + \frac{120-80}{2}\cos[(2)(145)] + 40\sin[(2)(145)] = \mathbf{69.25\ kN / m^3}$$

$$\tau_n = \frac{\sigma_y - \sigma_x}{2}\sin 2\theta - \tau_{xy}\cos 2\theta$$

$$= \frac{120-80}{2}\sin[(2)(145)] - 40\cos[(2)(145)] = \mathbf{-32.47\ kN / m^2}$$

9.2 a. $\sigma_x = 500$ lb / ft^2; $\sigma_y = 250$ lb / ft^2; $\tau_{xy} = -80$ lb / ft^2; $\theta = 45°$

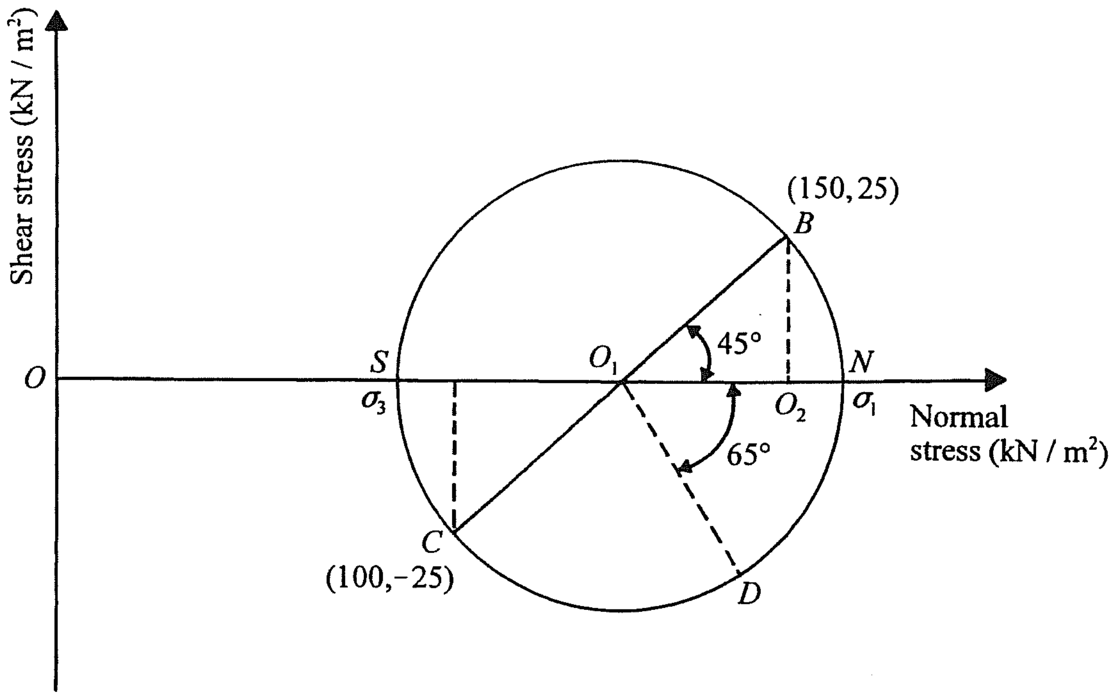
$$\left.\begin{matrix}\sigma_1\\ \sigma_3\end{matrix}\right\} = \frac{250+500}{2} \pm \sqrt{\left(\frac{250-500}{2}\right)^2 + (-80)^2}$$

$\sigma_1 =$ **523.4 lb / ft²**; $\sigma_3 =$ **226.6 lb / ft²**

b. $$\sigma_n = \frac{250+500}{2} + \frac{250-500}{2}\cos 90 - 80\sin 90 = \mathbf{295\ lb / ft^2}$$

c. $$\tau_n = \frac{250-500}{2}\sin 90 - (-80)\cos 90 = \mathbf{-125\ lb / ft^2}$$

9.3 a. The Mohr's circle is shown.



$$\overline{OO_1} = \frac{150+100}{2} = 125 \text{ kN/m}^2$$

$$\overline{O_1B} = \sqrt{\left(\frac{150-100}{2}\right)^2 + (25)^2} = 35.36 \text{ kN/m}^2$$

$\sigma_3 = \overline{OS} = 125 - 35.36 =$ **89.64 kN / m² (+)**

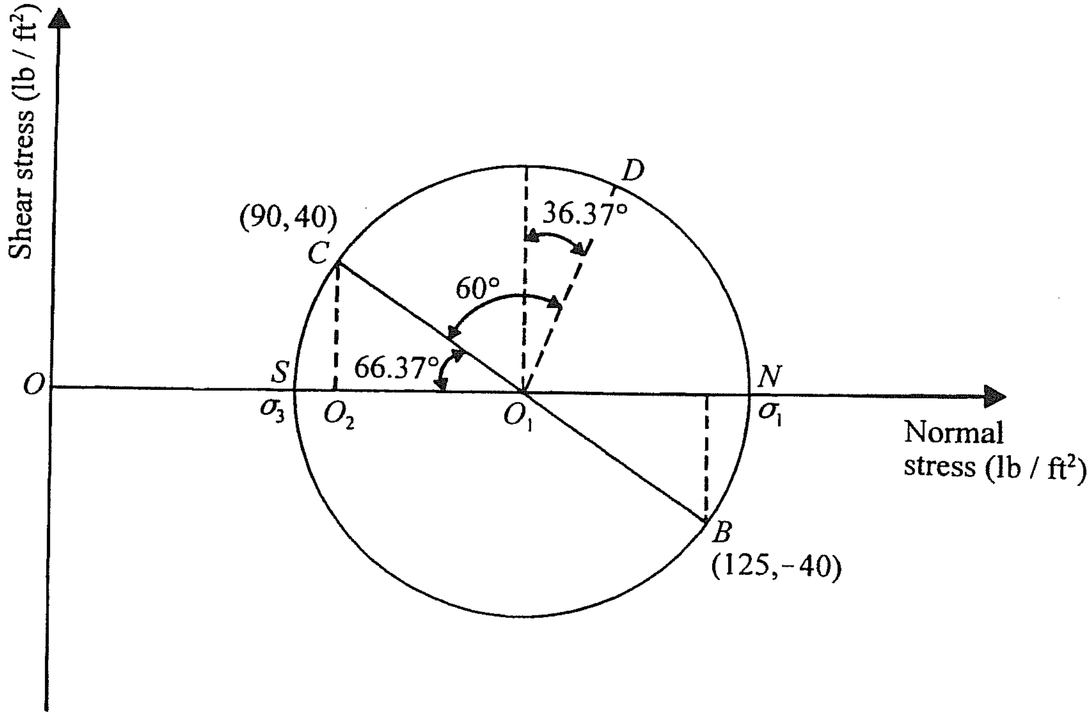
$\sigma_1 = \overline{ON} = 125 + 35.36 =$ **160.36 kN / m² (+)**

b. $\angle BO_1O_2 = \tan^{-1}\left(\frac{25}{25}\right) = 45°$

$\sigma_n = \overline{OO_1} + \overline{O_1D}\cos 65 = 125 + 35.36\cos 65 =$ **139.9 kN / m² (+)**

$\tau_n = \overline{O_1D}\sin 65 = 35.36\sin 65 =$ **32.05 kN / m² (−)**

9.4 a. The Mohr's circle is shown.



$$\overline{OO_1} = \frac{125+90}{2} = 107.5 \text{ lb/ft}^2$$

$$\overline{O_1O_2} = \frac{125-90}{2} = 17.5 \text{ lb/ft}^2$$

$$\overline{O_1B} = \sqrt{(17.5)^2 + (40)^2} = 43.66 \text{ lb/ft}^2$$

$\sigma_1 = \overline{ON} = 107.5 + 43.66 =$ **151.16 lb / ft²**

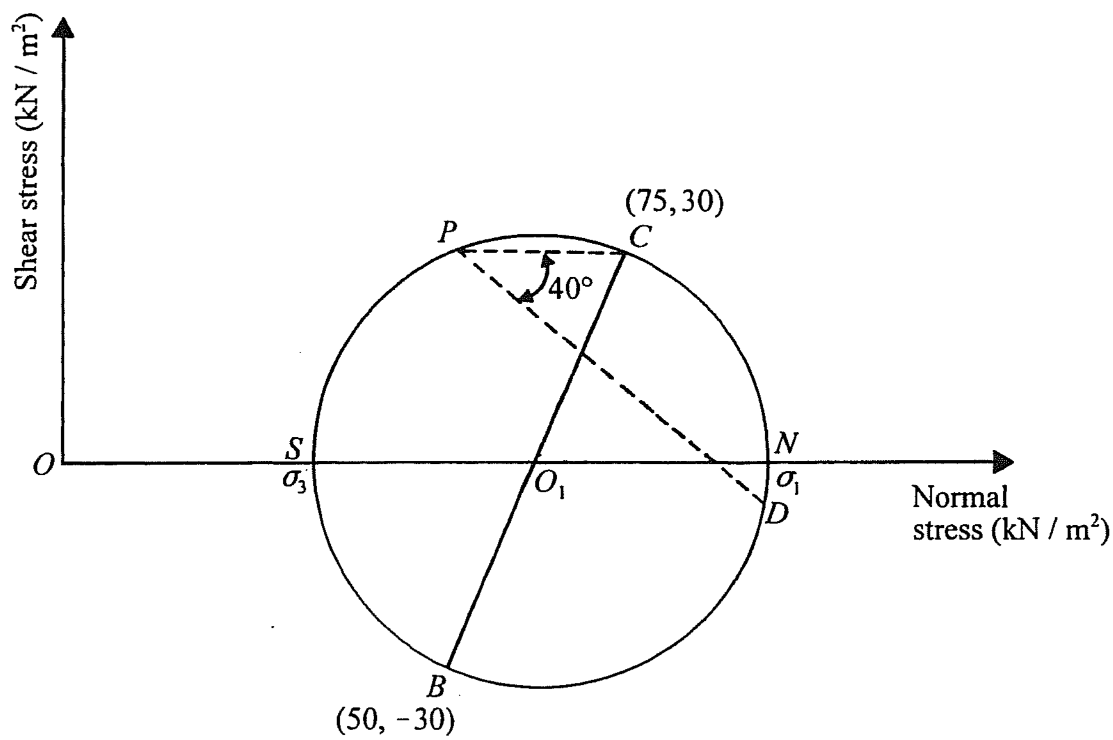
$\sigma_3 = \overline{OS} = 107.5 - 43.66 =$ **63.84 lb / ft²** (+)

b. $\angle CO_1O_2 = \tan^{-1}\left(\frac{40}{17.5}\right) = 66.37°$

$\sigma_n = \overline{OO_1} + \overline{O_1D}\sin(36.37) = 107.5 + 43.66\sin 36.37 =$ **133.4 lb / ft²**

$\tau_n = 43.66\cos 36.37 =$ **35.16 lb / ft²**

9.5 a. The Mohr's circle is shown.



$\sigma_1 = \overline{ON} = \mathbf{95\ kN/m^2}$; $\sigma_3 = \overline{OS} = \mathbf{30\ kN/m^2}$

b. σ_n and τ_n are coordinates of D. So

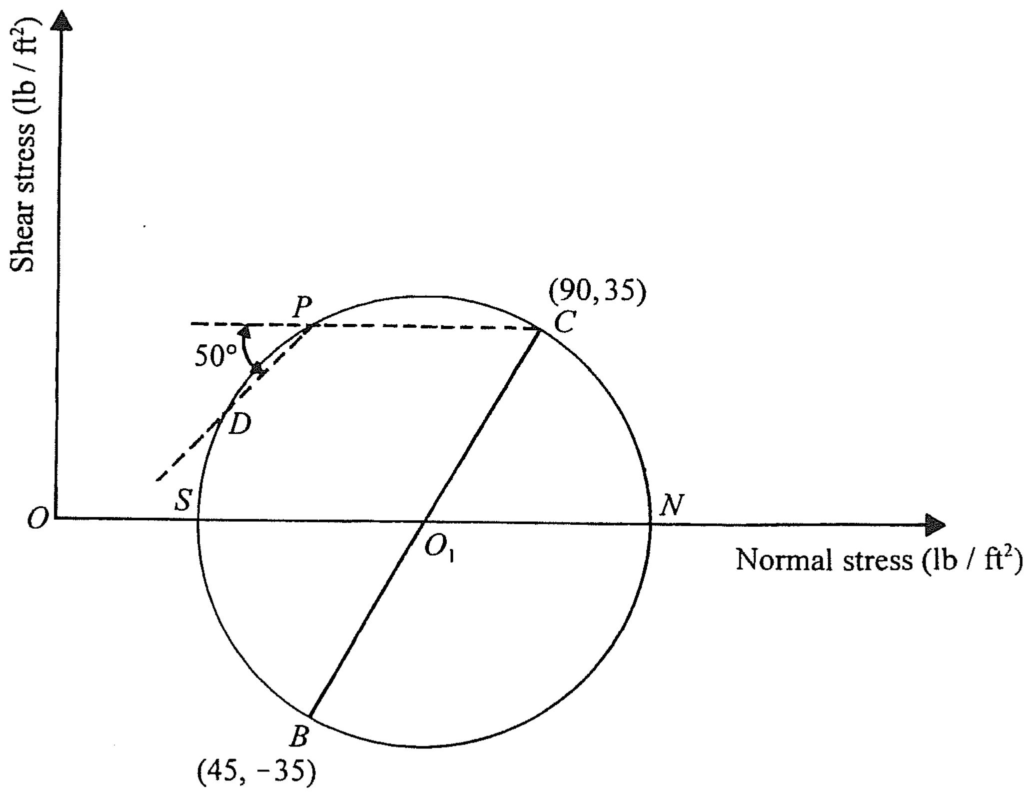
$\sigma_n \approx \mathbf{94.2\ kN/m^2}$; $\tau_n \approx \mathbf{7.1\ kN/m^2\ (-)}$

9.6 a. The Mohr's circle is shown on page 59.

$\sigma_1 = \overline{ON} \approx \mathbf{109.1\ kN/m^2}$; $\sigma_3 = \overline{OS} = \mathbf{25.9\ kN/m^2}$

b. σ_n and τ_n are coordinates of D. So

$\sigma_n \approx \mathbf{29.1\ kN/m^2}$; $\tau_n \approx \mathbf{16.08\ kN/m^2}$



Problem 9.6

9.7

Load @	P (lb)	r (ft)	z (ft)	$\frac{r}{z}$	I_1 (Table 9.1)	$\Delta\sigma_z = \frac{P}{z^2} I_1$ (lb / ft^2)
A	2000	$(10^2 + 5^2)^{0.5} = 11.18$	10	1.12	0.0626	1.25
B	4000	$(10^2 + 5^2)^{0.5} = 11.18$	10	1.12	0.0626	2.5
C	6000	5	10	0.5	0.2733	16.4

$\Delta\sigma_z = \Sigma 20.15$ lb / ft^2

9.8 Equation (9.16): $\eta = \sqrt{\frac{1-(2)(0.4)}{2-(2)(0.4)}} = 0.408$

Equation (9.15):

Load A: $\frac{(2000)(0.408)}{2\pi(10)^2}\left[\frac{1}{(0.408)^2 + (1.12)^2}\right]^{\frac{3}{2}} = 0.77 \text{ lb / ft}^2$

Load B: $\dfrac{(4000)(0.408)}{2\pi(10)^2}\left[\dfrac{1}{(0.408)^2+(1.12)^2}\right]^{\frac{3}{2}} = 1.53 \text{ lb/ft}^2$

Load C: $\dfrac{(6000)(0.408)}{2\pi(10)^2}\left[\dfrac{1}{(0.408)^2+(0.5)^2}\right]^{\frac{3}{2}} = 14.5 \text{ lb/ft}^2$

$\Delta\sigma_z = 0.77 + 1.53 + 14.5 = \mathbf{16.8\ lb/ft^2}$

9.9 Equation (9.19):

$$\Delta\sigma_z = \frac{2q_1z^3}{\pi\left[(x_1+x_2)^2+z^2\right]^2} + \frac{2q_2z^3}{\pi(x_2^2+z^2)^2+z^2} = \frac{(2)(100)(2)^3}{\pi(5^2+2^2)^2} + \frac{(2)(200)(2)^3}{\pi(2^2+2^2)^2}$$

$= \mathbf{16.53\ kN/m^2}$

9.10 $$\Delta\sigma_z = \frac{2q_1z^3}{\pi\left[(x_1+x_2)^2+z^2\right]^2} + \frac{2q_2z^3}{\pi(x_2^2+z^2)^2+z^2}$$

$$= \frac{(2)(100)(2.5)^3}{\pi\left[(3+2.5)^2+(2.5)^2\right]^2} + \frac{(2)(260)(2.5)^3}{\pi\left[(2.5)^2+(2.5)^2\right]^2}$$

$= \mathbf{20.83\ kN/m^3}$

9.11 $$\Delta\sigma_z = \frac{2q_1z^3}{\pi\left[(x_1+x_2)^2+z^2\right]^2} + \frac{2q_2z^3}{\pi(x_2^2+z^2)^2+z^2}$$

$$35 = \frac{(2)(750)(3)^3}{\pi(12^2+3^2)^2} + \frac{2q_2(3)^3}{\pi(4^2+3^2)^2} = 0.55 + 0.0275q_2$$

$q_2 = \mathbf{1252.7\ lb/ft}$

9.12 $\Delta\sigma_z$ at A due to $q_1 = \dfrac{2q_1z^2}{\pi\left(x^2+z^2\right)^2}$

or

$$(\Delta\sigma_z)_1 = \frac{(2)(100)(2)^3}{\pi\left[(2)^2+(2)^2\right]^2} = 7.96 \text{ kN/m}^2$$

Vertical component of $q_2 = q_2 \sin 45$

$$(\Delta\sigma_z)_2 = \frac{2q_2(\sin 45)z^3}{\pi\left[(5)^2+(2)^2\right]^2};\ (\Delta\sigma_z)_2 = 0.0043q_2$$

Horizontal component of $q_2 = q_2 \cos 45$

From Equation (9.21):

$$(\Delta\sigma_z)_3 = \frac{2q_2xz^2}{\pi\left(x^2+z^2\right)^2} = \frac{2q_2(\cos 45)(5)(2)^2}{\pi\left[(5)^2+(2)^2\right]^2} = 0.0107q_2$$

Total vertical stress,

$$\Delta\sigma_z = 10 \text{ kN/m}^2 = (\Delta\sigma_z)_1 + (\Delta\sigma_z)_2 + (\Delta\sigma_z)_3$$

$$10 = 7.96 + 0.0043q_2 + 0.0107q_2$$

$$q_2 = \frac{10-7.96}{0.015} = \mathbf{136\ kN/m}$$

9.13 $B = 10$ ft; $q = 200$ lb/ft^2; $x = 8$ ft; $z = 8$ ft

$$\frac{2x}{B} = \frac{(2)(8)}{10} = 1.6;\quad \frac{2z}{B} = \frac{(2)(8)}{10} = 1.6.\quad \text{From Table 9.4, } \frac{\Delta\sigma_z}{q} = 0.248$$

$$\Delta\sigma_z = (0.248)(200) = \mathbf{49.6\ lb/ft^2}$$

9.14 $\dfrac{2x}{B} = \dfrac{(2)(1.5)}{3} = 1;\quad \dfrac{2z}{B} = \dfrac{(2)(3)}{3} = 2.\quad$ From Table 9.4, $\dfrac{\Delta\sigma_z}{q} = 0.409$

$$\Delta\sigma_z = (60)(0.409) = \mathbf{24.54\ kN/m^2}$$

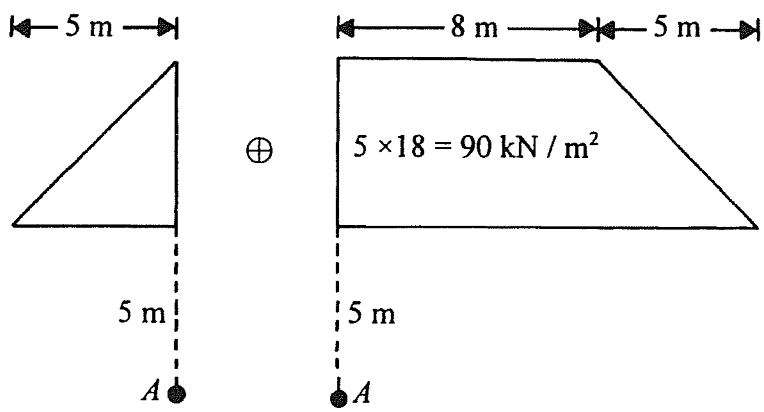
9.15 Equation (9.24):

$$\Delta\sigma_z = \frac{2Bqxz^2}{\pi\left\{\left[x^2 + z^2 - \left(\frac{B}{2}\right)^2\right]^2 + B^2z^2\right\}}$$

$$9 = \frac{(2)(1)(q)(1.5)(0.75)^2}{\pi\left\{\left[(1.5)^2 + (0.75)^2 - \left(\frac{1}{2}\right)^2\right]^2 + (1)^2(0.75)^2\right\}}$$

$q =$ **119.4 kN / m²**

9.16 Refer to the figure below.



With the notations given in Figure 9.17, for the left side:

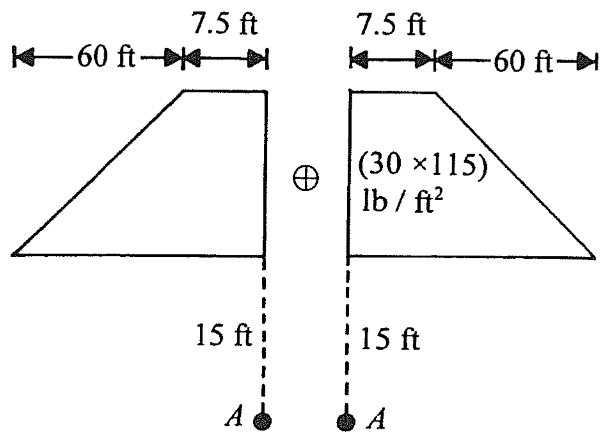
$\frac{B_1}{z} = \frac{0}{5} = 0;\ \ \frac{B_2}{z} = \frac{5}{5} = 1.$ From Figure 9.18, $I_{3(L)} = 0.24$

For the right side,

$\frac{B_1}{z} = \frac{8}{5} = 1.6;\ \ \frac{B_2}{z} = \frac{5}{5} = 1.$ From Figure 9.18, $I_{3(R)} = 0.48$

$\Delta\sigma_z = q[I_{3(L)} + I_{3(R)}] = (90)(0.24 + 0.48) =$ **64.8 kN / m²**

9.17 At A:



For the left side:

$$\frac{B_1}{z}=\frac{7.5}{15}=0.5$$

$$\frac{B_2}{z}=\frac{60}{15}=4$$

$$I_3=0.468$$

For the right side:

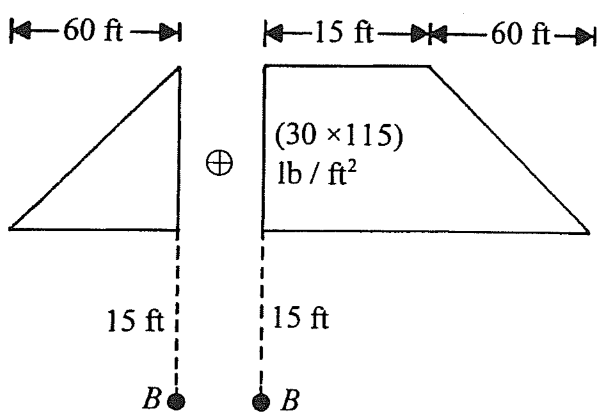
$$\frac{B_1}{z}=0.5$$

$$\frac{B_2}{z}=4$$

$$I_3=0.468$$

$\Delta\sigma_z=(30)(115)(0.468+0.468)\approx$ **3229 lb / ft²**

At B:



For the left side:

$$\frac{B_1}{z}=\frac{0}{15}=0$$

$$\frac{B_2}{z}=\frac{60}{15}=4$$

$$I_3=0.42$$

For the right side:

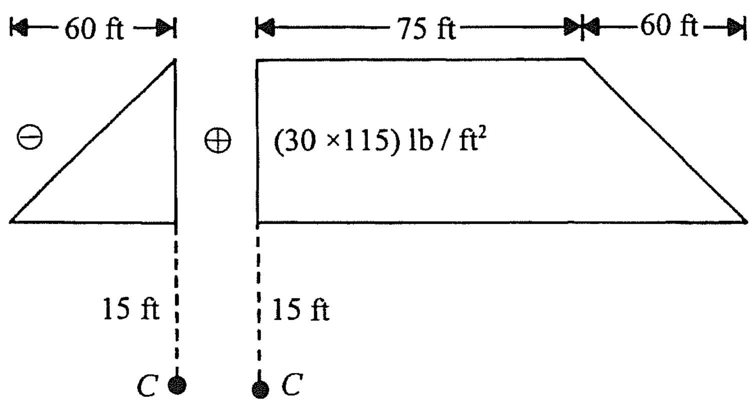
$$\frac{B_1}{z}=\frac{15}{15}=1$$

$$\frac{B_2}{z}=\frac{60}{15}=4$$

$$I_3=0.48$$

$\Delta\sigma_z=(30)(115)(0.42+0.48)\approx$ **3105 lb / ft²**

At C:



For the left side:

$$\frac{B_1}{z} = 0$$

$$\frac{B_2}{z} = \frac{60}{15} = 4$$

$$I_3 = 0.42$$

For the right side:

$$\frac{B_1}{z} = \frac{75}{15} = 5$$

$$\frac{B_2}{z} = \frac{60}{15} = 4$$

$$I_3 = 0.5$$

$$\Delta\sigma_z = (30)(115)(0.5 - 0.42) \approx \mathbf{276\ lb/ft^2}$$

9.18 Equation (9.30) and Table 9.6: $q = 3500\ \text{lb/ft}^2$

R (ft)	z (ft)	$\frac{z}{R}$	$\frac{\Delta\sigma_z}{q}$	$\Delta\sigma_z$ (lb / ft²)
6	1.5	0.4	0.9488	**3321**
6	3	0.5	0.9106	**3187**
6	6	1.0	0.6465	**2263**
6	9	1.5	0.4240	**1484**
6	12	2.0	0.2845	**996**

9.19 Equation (9.31) and Tables 9.7 and 9.8: $q = 300 \text{ kN/m}^2$

z (m)	r (m)	R (m)	$\frac{z}{R}$	$\frac{r}{R}$	A'	B'	$\Delta\sigma_z$ (kN/m²)
4.8	0	4	1.2	0	0.23178	0.31485	**164.0**
4.8	0.8	4	1.2	0.2	0.22795	0.30730	**160.6**
4.8	1.6	4	1.2	0.4	0.21662	0.28481	**150.4**
4.8	4.0	4	1.2	1.0	0.15101	0.14915	**90.1**
4.8	6.0	4	1.2	1.5	0.09192	0.04378	**40.7**
4.8	8.0	4	1.2	2.0	0.05260	0.00023	**15.8**

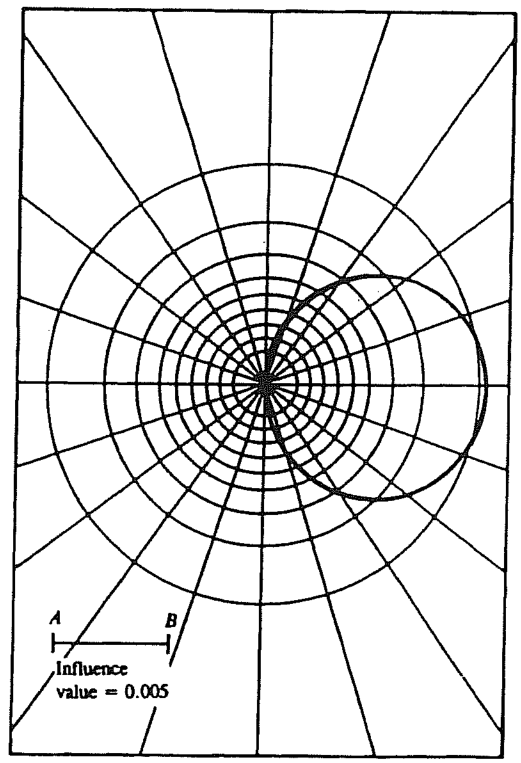
9.20 Refer to the Newmark's chart.

The plan is drawn to scale.

$\overline{AB} = 4$ m. $M \approx 65$.

$\Delta\sigma_z = (IV)qM = (0.005)(300)(65)$

$= \mathbf{97.5 \text{ kN/m}^2}$

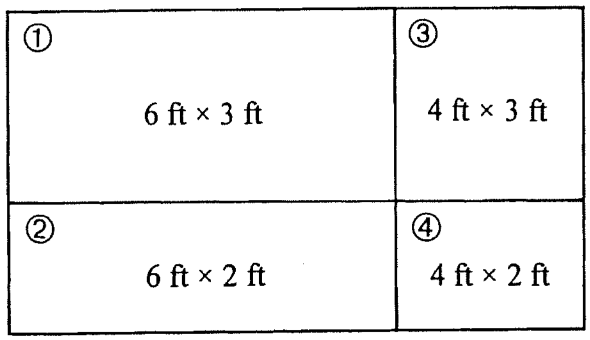


9.21 a. Equations (9.36) and (9.37): $n = \frac{L}{z} = \frac{10}{5} = 2; \quad m = \frac{B}{z} = \frac{5}{5} = 1$

Equation (9.34): $\Delta\sigma_z = qI_4$; $I_4 = 0.1999$

$\Delta\sigma_z = (1800)(0.1999) = \mathbf{359.8 \text{ lb/ft}^2}$

b. Refer to the figure below.



For rectangle 1: $m = \frac{3}{5} = 0.6;\quad n = \frac{6}{5} = 1.2;\quad I_4 = 0.1431$

For rectangle 2: $m = \frac{2}{5} = 0.4;\quad n = \frac{6}{5} = 1.2;\quad I_4 = 0.1063$

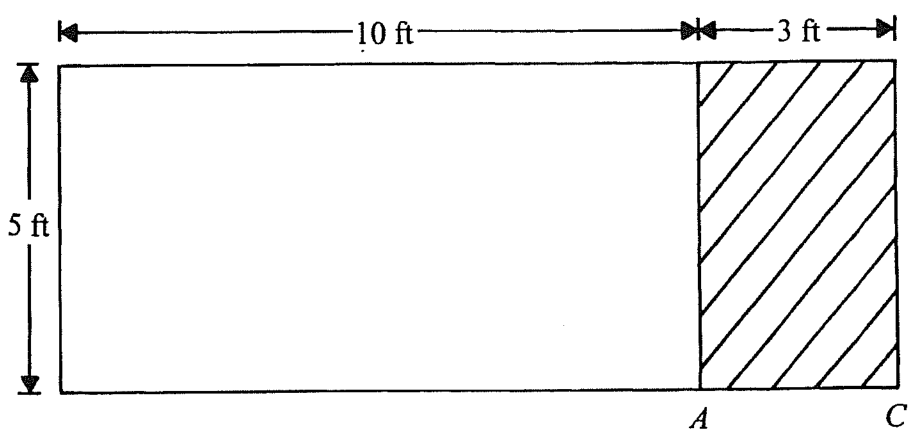
For rectangle 3: $m = \frac{3}{5} = 0.6;\quad n = \frac{4}{5} = 0.8;\quad I_4 = 0.1247$

For rectangle 4: $m = \frac{2}{5} = 0.4;\quad n = \frac{4}{5} = 0.8;\quad I_4 = 0.0931$

$$\Delta\sigma_z = q[I_{4(1)} + I_{4(2)} + I_{4(3)} + I_{4(4)}] = (1800)(0.1431 + 0.1063 + 0.1247 + 0.0931)$$

$$= \mathbf{841\ lb/ft^2}$$

c. Refer to the figure.



$$\Delta\sigma_z = \left\langle \begin{array}{l} \text{stress at } C \text{ due to rectangular area 13 ft} \times \text{5 ft} \\ \quad - \text{ stress at } C \text{ due to rectangular area 3 ft} \times \text{5 ft} \end{array} \right.$$

For rectangular area 13 ft × 5 ft: $m = \frac{5}{5} = 1;\quad n = \frac{13}{5} = 2.6;\quad I_4 = 0.202$

For rectangular area 3 ft × 5 ft: $m = \frac{3}{5} = 0.6;\quad n = \frac{5}{5} = 1;\quad I_4 = 0.1361$

$$\Delta\sigma_z = q(0.202 - 0.1361) = (1800)(0.202 - 0.1351) = \mathbf{118.6\ lb / ft^2}$$

9.22 Equations (9.41), (9.42), and (9.43):

$$b = \frac{B}{2} = \frac{5}{2} = 2.5 \text{ ft}$$

$$m_1 = \frac{L}{B} = \frac{10}{5} = 2$$

$$n_1 = \frac{z}{b} = \frac{15}{2.5} = 6$$

From Table 9.10, $I_5 = 0.095$

$$\Delta\sigma_z = qI_5 = (1800)(0.095) = \mathbf{171\ kN / m^2}$$