Question 1

A 208-V four-pole 60-Hz Y-connected induction motor has the following parameters:
Armature resistance 0.22 Ω, stator leakage reactance 0.43 Ω, magnetizing reactance 15 Ω, rotor resistance 0.127 Ω and rotor leakage reactance 0.43 Ω (the latter two are referred to the stator side). Furthermore, all losses except copper losses can be neglected.

a) Draw the single-phase equivalent circuit of the motor and label all its components with values from above.

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\[ I_A \quad R_1 \quad jX_1 \quad R_2 \quad jX_2 \]
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\[
V_\phi \quad j15 \Omega \quad jX_M \quad R_2 \left( \frac{1-s}{s} \right)
\]

or

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+ \quad I_1 \quad jX_1 \quad R_1 \quad \times \quad jX_2 \quad I_2 \quad +
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V_o \quad jX_M \quad \times \quad E_1 \quad +
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For a speed of 1710 rpm find

b) The slip s

\[
n_{Sync} = \frac{120 f}{P} = \frac{120 \cdot 60}{4} = 1800 \text{ rpm} \Rightarrow s = \frac{n_{Sync} - n}{n_{Sync}} = \frac{1800 - 1710}{1800} = 0.05
\]

c) The line current \( I_L \) and power factor

\[
V_\phi = \frac{208}{\sqrt{3}} = 120 V,
\]

\[
Z(s) = R_1 + jX_1 + jX_M \frac{R_2}{s} + jX_2 + jX_M = (2.557 + j1.233) \Omega = 2.839\Omega \angle 25.7^\circ
\]

\[
I_1 = I_A = I_L = \frac{V_\phi}{Z(s)} = \frac{120\angle 0^\circ}{2.839\angle 25.7^\circ} = 42.3 A \angle -25.7^\circ, \quad PF = \cos(-25.7^\circ) = 0.9 \text{ lagging}
\]

It is also possible to work with the equivalent circuit (like problem 7-7 in book):
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\[ Z_F = R_F + jX_F = jX_M \frac{R_2/s + jX_2}{R_2/s + jX_2} = (2.337 + j0.803)\Omega = 2.471\Omega \angle 19^\circ \] with

\[ Z(s) = R_i + jX_i + Z_F \]

d) The output power \( P_{\text{OUT}} \) and the overall efficiency \( \eta \)

\[ I_M = \frac{E_1}{jX_M} = \frac{V_\phi - I_1(R_i + jX_i)}{jX_M} = \frac{120 - 42.3e^{-j25.7^\circ}}{0.22 + j0.43} = -0.823 + j6.915, \quad A = 6.96 A \angle 96.8^\circ \]

\[ I_2 = I_1 - I_M = 42.3e^{-j25.7^\circ} - 6.96e^{-j96.8^\circ} = (38.9 - j11.5) A = 40.6 A \angle -16.4^\circ \]

\[ P_{\text{conv}} = 3I_2^2R_2 \frac{1-s}{s} = 3 \cdot 40.6^2 \cdot 0.127 \cdot 0.95 = 11.93 kW \]

since all losses such as core, windage, and miscellaneous losses are neglected

\[ P_{\text{out}} = P_{\text{conv}} = 11.93 kW \]

\[ P_{\text{in}} = 3V_\phi I_L \cdot PF = 3 \cdot 120 \cdot 42.3 \cdot 0.9 = 13.7 kW \]

\[ \eta = \frac{P_{\text{out}}}{P_{\text{in}}} \cdot 100\% = \frac{11.93}{13.7} \cdot 100\% = 87.1\% \]

**Question 2**

A 3-phase induction motor is connected to the end of a 1.5 mile long overhead transmission line. The motor draws 2.74 MVA at 4.16 kV with a lagging power factor of 0.75 from the line. A Y-connected 4 MVA synchronous generator with a synchronous reactance of 1pu (the stator resistance can be neglected) powers the sending end of the line. The phase conductors of this line are made entirely of aluminum and are arranged in an equilateral triangle 3.3 ft apart.

a) Draw the complete single-phase equivalent circuit of this problem, annotate the unknown line impedances, and calculate the load impedance, the load voltage, the load current, and the generator impedance. Enter their values into the equivalent circuit diagram.

\[ Z_L = \frac{4.16^2}{2.74} = 6.31\Omega, \quad \phi_L = \cos^{-1}(0.75) = 41.41^\circ, \quad I_{\text{Load}} = \frac{2.4 \cdot 10^3 \angle 0^\circ}{6.31 \angle 41.41^\circ} = 380.3 A \angle -41.41^\circ, \]

\[ X_S = j \frac{4.16^2}{4} = j4.33\Omega \]

\[ \text{G} \quad \text{E}_A \quad \text{V}_{\text{send}} \quad \text{V}_{\text{load}} = 2.4 \text{kV} \angle -0^\circ \quad \text{Load} \]

\[ \text{Z}_L = 6.31\Omega \angle 41.41^\circ \]

\[ \text{R}_T \quad \text{I}_{\text{load}} = 380.3 A \angle -41.41^\circ \]

\[ \text{X}_T \]

\[ \text{G} \quad \text{TL} \]
b) Size the aluminum conductors for the load current above assuming an ambient temperature of 40°C with wind blowing and with a conductor temperature of 75°C (use tables below).

In order to carry 380.3 A of current a conductor of size 300 kcmil is necessary.

c) Calculate the impedance of the transmission line in Ohm (use tables below).

Hint: The reactance in Ohm per mile is $X' = 0.2794 \frac{f}{60} \log \frac{GMD}{GMR}$ and 1 mile = 5280 ft
The 300 kcmil conductor has a geometric mean radius GMR of 0.0198 ft and resistance of 0.0705 mΩ/ft. Since the conductor arrangement is totally symmetric the geometric mean distance GMD = 3.3 ft.

\[
X_{rL} = 1.5 \cdot 0.2794 \cdot \frac{60}{60} \log \frac{3.3}{0.0198} = 0.93 \Omega, \quad R_{rL} = 1.5 \cdot 5280 \cdot 0.0705 \cdot 10^{-3} = 0.56 \Omega
\]

d) For the load above what is the induced voltage in the generator?

\[
E_A = 2.4 \cdot 10^3 + 380.3 e^{-j41.4^\circ} \cdot (0.56 + j0.93 + j4.33) = (3.88 + j1.36) kV = 4.11 kV \angle 19.3^\circ
\]

**Question 3**

The capacitance between conductors of a transmission line is considered in (circle the correct answer)

- the “short line model”
- the “medium length line model”
- both

**Question 4**

Increasing the field current in a synchronous motor which is driving a constant torque load and is connected to a constant frequency source will change the motor’s (circle the correct answer)

- real power
- reactive power
- both