Induced Voltage in Stator Coil – P-Poles

- For a P-pole machine the flux per pole remains the same (integrating $d\phi = r l B_m \cos(P/2) d\alpha$ over one pole pitch $-\pi/P$ to $+\pi/P$)
  \[ \phi = 2rlB_m \]
- The electrical frequency becomes
  \[ \omega_{sl} = \frac{P}{2} \omega_m = \omega \]
- Therefore, the induced voltage per pole-pair (one sub-winding) is
  \[ e_{\text{ind, pole}} = N_c \phi_m \cos\left(\frac{P}{2} \omega_m t\right) \]
- The entire phase winding consists of $P/2$ sub-windings connected in series resulting in the total induced phase voltage
  \[ e_{\text{ind}} = \frac{P}{2} N_c \phi_m \cos\left(\frac{P}{2} \omega_m t\right) \]

Example 4-2 (Book)

- Additional question 2
  - Assuming the same field from the rotor, a rated current of $I=10 \text{ A (RMS)}$ in the stator at PF=0.8, a stator magnetic field of 0.4 mT/A-turn, and a stator resistance of $0.3 \Omega$ per phase, calculate the torque constant ($k$ in Nm/T²) in
    \[ \tau_{\text{ind}} = k B_r B_s \sin(\gamma) \]
    if the angle between the rotor field and the stator field space vectors is $\gamma = -100^\circ$ (assume $k > 0$).
  - What is the direction of rotation (CW, CCW) if this is a motor?
Example 4-2 (Book)

- Additional question 2

\[ P_{el} = \sqrt{3} VI \cdot PF = \sqrt{3} \cdot 208 \cdot 10 \cdot 0.8 = 2882 \, W \]

\[ P_{Loss,R} = 3 I^2 R = 3 \cdot 10^2 \cdot 0.3 = 90 \, W \]

\[ \omega_m \tau_{ind} = P_{conv} \rightarrow \tau_{ind} = \frac{60 (P_{el} - P_{Loss,Cu})}{2\pi n} = \frac{60 \cdot 2.792}{2\pi 3600} = 7.41 \, Nm \]

\[ B_R = 0.2 \, T \]

\[ \tau \] is CW (see x-product) and therefore negative with respect to the positive mathematical counting direction (i.e. CCW). The speed must also be CW since this is a motor.

\[ B_S = 0.4 \cdot 10^{-4} \cdot 15 \sqrt{2} \cdot 10 \]

\[ = 0.085 \, T \]

\[ k = \frac{\tau_{ind}}{B_R B_S \sin(\gamma)} = \frac{-7.41}{0.2 \cdot 0.085 \sin(-100^\circ)} = 442.6 \frac{Nm}{T^2} \]

Comments on Power, Torque, and Speed on the Shaft of Rotating Machines

Rated power
Mot: at shaft
Gen: at terminals

\[ \bar{\tau} = k \bar{B}_R \times \bar{B}_S \]

Positive z-direction

Motor supplies power to load
\[ P_{Shaft} = +|\omega_M||\tau_{Mot}| > 0 > 0 \]

Prime mover
\[ P_{Shaft} = -|\omega_M||\tau_{Mover}| > 0 < 0 \]

Generator absorbs power from prime mover
\[ P_{Shaft} = +|\omega_M||\tau_{Gen}| > 0 > 0 \]

Attempts to accelerate CCW

For \(\omega_M=\text{const} \) \( (d\omega_M/dt=0) \) the sum of the torques at the shaft connection must be zero.
Synchronous Machines (SM)

- The majority of generators are of the synchronous machine type
- The rotor provides permanent magnetic excitation
  - Permanent magnet
  - Electromagnet (DC)
- The speed of the shaft is always in synchronism with the electrical frequency

SM Rotor Construction

Pole shape provides sinusoidal $B_R$ distribution

Also possible: round rotor (no saliency)
SM Construction

- DC excitation system
- Rotor poles with DC field winding
- Stator iron to close magnetic path and slots for AC windings
- AC stator windings
- Cooling fan
- Bearings
- Shaft stub for torque application

SM Modeling for Steady State Operation

- **Purpose**: describing the behavior of the SM under different steady state load conditions
- **The model must account for the**
  - Resistance of the stator (armature) winding
  - Self-inductance (reactance) of the armature winding
  - Effect of the armature B-field on the total B-field (armature reaction)
- **Assumptions, simplifications**
  - Balanced conditions
  - No spatial distortions of the B-field (infinite number of slots, perfect pole shapes), i.e. “perfect” sinusoidal B-field distribution in the air gap
SM No-Load Characteristic

Rotor flux magnitude \( \phi \)

Induced stator voltage magnitude \( E_A \)

\[ E_A = k \phi \omega \]

Effect of saturation: for large field currents the no-load armature voltage magnitude scales sub-proportionally with \( I_F \) and the voltage waveform becomes non sinusoidal (distorted).