Problem 9-6 refers to a single-phase, 8 kV, 50 Hz, 50 km long transmission line consisting of two aluminum conductors with a 3 cm diameter separated by spacing of 2 m.

9.6. The single-phase transmission line is operating with the receiving side of the line open-circuited. The sending end voltage is 8 kV at 50 Hz. How much charging current is flowing in the line? Additional question: What is the voltage on the receiving end?

Tip: Use the procedure outlined in the book for the solution (no tables).

**SOLUTION** Although this line is in the "short" range of lengths, we will treat it as a medium-length line, because we must include the capacitances if we wish to calculate charging currents. The appropriate transmission line model is shown below.

![Transmission Line Model](image)

The charging current can be calculated by open-circuiting the output of the transmission line and calculating $I_s$:

$$I_s = \frac{YV_s}{2} + \frac{V_s}{Z + \frac{1}{Y/2}}$$

$$I_s = \frac{\left(8.95 \times 10^{-5} \text{ S}\right)(8000 \angle 0^\circ \text{ V})}{2} + \frac{8000 \angle 0^\circ \text{ V}}{(2.1 + j32.3 \Omega) + \frac{1}{f8.95 \times 10^{-5} \text{ S/2}}}$$

$$I_s = 0.358 \angle 90^\circ \text{ A} + 0.358 \angle 90^\circ \text{ A} = 0.716 \angle 90^\circ \text{ A}$$

$$V_s = I_s(R + j\omega L) + V_R$$

$$V_R = 8k - 0.716 \angle 90^\circ(100 + j31.4) = 8022 \angle -0.5^\circ$$
9-11. A 138 kV, 200 MVA, 60 Hz, three-phase, power transmission line is 100 km long, and has the following characteristics:

\[ r = 0.103 \, \Omega/\text{km} \]
\[ x = 0.525 \, \Omega/\text{km} \]
\[ y = 3.3 \times 10^{-6} \, \text{S/km} \]

(a) What is per phase series impedance and shunt admittance of this transmission line?
(b) Should it be modeled as a short, medium, or long transmission line?
(c) Calculate the \( A\text{BCD} \) constants of this transmission line.
(d) Sketch the phasor diagram of this transmission line when the line is supplying rated voltage and apparent power at a 0.90 power factor lagging.
(e) Calculate the sending end voltage if the line is supplying rated voltage and apparent power at 0.90 PF lagging.
(f) What is the voltage regulation of the transmission line for the conditions in (e)?
(g) What is the efficiency of the transmission line for the conditions in (e)?

**SOLUTION**

(a) The per-phase series impedance of this transmission line is

\[ Z = (0.103 + j0.525 \, \Omega/\text{km})(100 \, \text{km}) = 10.3 + j52.5 \, \Omega \]

The per-phase shunt admittance of this transmission line is

\[ Y = (j3.3 \times 10^{-6} \, \text{S/km})(100 \, \text{km}) = j0.00033 \, \text{S} \]

(b) This transmission line should be modeled as a medium length transmission line.

(c) The \( A\text{BCD} \) constants for a medium length line are given by the following equations:

\[ A = \frac{ZY}{2} + 1 \quad B = Z \]
\[ C = Y \left( \frac{ZY}{4} + 1 \right) \quad D = \frac{ZY}{2} + 1 \]
\[ A = \frac{ZY}{2} + 1 = \frac{(10.3 + j52.5 \, \Omega)(j0.00033 \, \text{S})}{2} + 1 = 0.9913 \angle 0.1^\circ \]
\[ B = Z = 10.3 + j52.5 \, \Omega = 53.5 \angle 78.9^\circ \, \Omega \]
\[ C = Y \left( \frac{ZY}{4} + 1 \right) = (j0.00033 \, \text{S}) \left[ \frac{(10.3 + j52.5 \, \Omega)(j0.00033 \, \text{S})}{4} + 1 \right] \]
\[ C = 3.286 \times 10^{-4} \angle 90^\circ \, \text{S} \]
\[ D = \frac{ZY}{2} + 1 = \frac{(10.3 + j52.5 \, \Omega)(j0.00033 \, \text{S})}{2} + 1 = 0.9913 \angle 0.1^\circ \]
(d) The phasor diagram is shown below:

![Phasor Diagram]

(e) The rated line voltage is 138 kV, so the rated phase voltage is $138 \text{ kV} / \sqrt{3} = 79.67 \text{ kV}$, and the rated current is

$$I_L = \frac{S_{\text{rated}}}{\sqrt{3}V_{LL}} = \frac{200,000,000 \text{ VA}}{\sqrt{3}(138,000 \text{ V})} = 837 \text{ A}$$

If the phase voltage at the receiving end is assumed to be at a phase angle of $0^\circ$, then the phase voltage at the receiving end will be $V_R = 79.67 \angle 0^\circ \text{ kV}$, and the phase current at the receiving end will be $I_R = 837 \angle -25.8^\circ \text{ A}$. The current and voltage at the sending end of the transmission line are given by the following equations:

$$V_s = AV_R + BI_R$$
$$V_s = (0.9913 \angle 0.1^\circ)(79.67 \angle 0^\circ \text{ kV}) + (53.5 \angle 78.9^\circ \text{ k}\Omega)(837 \angle -25.8^\circ \text{ A})$$
$$V_s = 111.8 \angle 18.75^\circ \text{ kV}$$

$$I_s = CV_R + DI_R$$
$$I_s = (3.286 \times 10^{-4} \angle 90^\circ \text{ S})(79.67 \angle 0^\circ \text{ kV}) + (0.9913 \angle 0.1^\circ)(837 \angle -25.8^\circ \text{ A})$$

$$I_s = 818.7 \angle -24.05^\circ \text{ A}$$

(f) The voltage regulation of the transmission line is

$$VR = \frac{V_s - V_R}{V_R} \times 100\% = \frac{111.8 \text{ kV} - 79.67 \text{ kV}}{79.67 \text{ kV}} \times 100\% = 40.3\%$$

(g) The output power from the transmission line is

$$P_{\text{out}} = S \cdot \text{PF} = (200 \text{ MVA}) \cdot (0.9) = 180 \text{ MW}$$

The input power to the transmission line is

$$P_{\text{in}} = 3V_{s}I_{\phi} \cos \theta = 3(111.8 \text{ kV})(818.7 \text{ A}) \cos (42.8^\circ) = 201.5 \text{ MW}$$

The resulting efficiency is

$$\eta = \frac{P_{\text{in}}}{P_{\text{out}}} \times 100\% = \frac{180 \text{ kW}}{201.5 \text{ kW}} \times 100\% = 89.3\%$$
9-15. The transmission line of Problem 9-11 is connected between two infinite buses, as shown in Figure P9-1. Answer the following questions about this transmission line.

Figure P9-1  A three-phase transmission line connecting two infinite busses together.

(a) If the per-phase (line-to-neutral) voltage on the sending infinite bus is 80∠10° kV and the per-phase voltage on the receiving infinite bus is 76∠0° kV, how much real and reactive power are being supplied by the transmission line to the receiving bus?

(b) If the per-phase voltage on the sending infinite bus is changed to 82∠10° kV, how much real and reactive power are being supplied by the transmission line to the receiving bus? Which changed more, the real or the reactive power supplied to the load?

(c) If the per-phase voltage on the sending infinite bus is changed to 80∠15° kV, how much real and reactive power are being supplied by the transmission line to the receiving bus? Compared to the conditions in part (a), which changed more, the real or the reactive power supplied to the load?

(d) From the above results, how could real power flow be controlled in a transmission line? How could reactive power flow be controlled in a transmission line?

Tip: Neglect the shunt admittance for this problem (although the line is longer than 50 mi).

SOLUTION

(a) If the shunt admittance of the transmission line is ignored, the relationship between the voltages and currents on this transmission line is

$$V_S = V_R + RI + jXI$$

where $I_S = I_R = I$. Therefore we can calculate the current in the transmission line as

$$I = \frac{V_S - V_R}{R + jX}$$

$$I = \frac{80,000\angle 10^\circ - 76,000\angle 0^\circ}{10.3 + j52.5 \Omega} = 265 \angle -0.5^\circ \text{ A}$$

The real and reactive power supplied by this transmission line is

$$P = 3V_{\phi,R}I_{\phi} \cos \theta = 3(76 \text{ kV})(265 \text{ A}) \cos (0.5^\circ) = 60.4 \text{ MW}$$

$$Q = 3V_{\phi,R}I_{\phi} \sin \theta = 3(76 \text{ kV})(265 \text{ A}) \sin (0.5^\circ) = 0.53 \text{ MVAR}$$

(b) If the sending end voltage is changed to 82∠10° kV, the current is

$$I = \frac{82,000\angle 10^\circ - 76,000\angle 0^\circ}{10.3 + j52.5 \Omega} = 280 \angle -7.7^\circ \text{ A}$$

The real and reactive power supplied by this transmission line is

$$P = 3V_{\phi,R}I_{\phi} \cos \theta = 3(76 \text{ kV})(280 \text{ A}) \cos (7.7^\circ) = 63.3 \text{ MW}$$

$$Q = 3V_{\phi,R}I_{\phi} \sin \theta = 3(76 \text{ kV})(280 \text{ A}) \sin (7.7^\circ) = 8.56 \text{ MVAR}$$

In this case, there was a relatively small change in $P$ (3 MW) and a relatively large change in $Q$ (8 MVAR) supplied to the receiving bus.
(c) If the sending end voltage is changed to $82 \angle 10^\circ$ kV, the current is
\[ I = \frac{80,000 \angle 15^\circ - 76,000 \angle 0^\circ}{10.3 + j52.5 \ \Omega} = 388 \angle 7.2^\circ \text{ A} \]

The real and reactive power supplied by this transmission line is
\[ P = 3V_{\phi,2}I_{\phi}\cos \theta = 3(76 \text{ kV})(388 \text{ A})\cos(-7.2^\circ) = 87.8 \text{ MW} \]
\[ Q = 3V_{\phi,2}I_{\phi}\sin \theta = 3(76 \text{ kV})(388 \text{ A})\sin(-7.2^\circ) = -11 \text{ MVAR} \]

In this case, there was a relatively large change in $P$ (27.4 MW) and a relatively small change in $Q$ (11.5 MVAR) supplied to the receiving bus.

(d) From the above results, we can see that real power flow can be adjusted by changing the phase angle between the two voltages at the two ends of the transmission line, while reactive power flow can be changed by changing the relative magnitude of the two voltages on either side of the transmission line.