1. A 3-phase, 4-pole synchronous machine rotates CCW with 1500 rpm and produces a rated torque of 668.45 Nm at the shaft in CW direction. The total rated losses are 5 kW and the rated stator (copper) losses alone are 3 kW. The rated terminal current is measured to be 150.4 A with a PF = 0.8 lagging.

a) Determine the missing electrical ratings of the machine (voltage, frequency, apparent power, and efficiency) and the induced torque (transferred between rotor and stator). Is a generator or a motor? Draw the power flow diagram of this machine (with actual values of the power included in the diagram).

b) Size a transformer for this machine (voltage rating and winding configuration) which steps the line current down to 17.35 A. Chose a winding configuration that yields the smallest possible current in the windings of the transformer.

Solution

a) Since the torque is in opposite direction of the rotation this is a generator. The electrical frequency of a $P = 4$ pole synchronous machine becomes

$$ f = \frac{P \omega_m}{2 \pi} = \frac{4 \cdot 157.08}{2 \pi} = 50 \text{ Hz} \quad \text{with} \quad \omega_m = 2 \pi \frac{1500}{60} = 157.08 \text{ rad/s} $$

The power flowing into the shaft is $P_{shaft} = \omega_m \tau_{shaft} = 668.45 \cdot 157.08 = 105 \text{ kW}$. With the total losses of 5 kW the electrical output power becomes

$$ P_{out} = P_{shaft} - P_{loss} = 105 - 5 = 100 \text{ kW} \ . $$

With 2 kW of losses in the rotor alone (5 kW total minus the 3 kW stator losses) the power transferred from the rotor to the stator becomes $P_{ind} = 103 \text{ kW}$. Therefore, the induced torque becomes

$$ \tau_{ind} = \frac{P_{ind}}{\omega_m} = \frac{103}{157.08} = 655.72 \text{ Nm} \ .$$

The voltage is calculated from the apparent output power

$$ S_{out} = \sqrt{3} V_L I_L = \frac{P_{out}}{PF} = \frac{100}{0.8} = 125 \text{ kVA} \rightarrow V_L = \frac{S_{out}}{\sqrt{3} I_L} = \frac{125 \cdot 10^3}{\sqrt{3} \cdot 150.4} = 480 \text{ V} $$

The efficiency is simply

$$ \eta = \frac{P_{out}}{P_{in}} \times 100\% = \frac{100}{105} \times 100\% = 95.2\% $$
b) Stepping down the line current means stepping up the voltage by the inverse ratio, i.e.
\[
\frac{V_1}{V_2} = \frac{I_2}{I_1} \rightarrow V_2 = V_1 \frac{I_1}{I_2} = 480 \frac{150.4}{17.35} = 4.16 \text{kV}.
\]
In order to yield the smallest winding current we chose the \(\Delta\Delta\) configuration. Therefore, the transformer will have the ratings 100 MVA, 480/4160 V, 60 Hz, \(\Delta\Delta\)

Solution

5-1)
At a location in Europe, it is necessary to supply 300 kW of 60-Hz power. The only power sources available operate at 50 Hz. It is decided to generate the power by means of a motor-generator set consisting of a synchronous motor driving a synchronous generator. How many poles should each of the two machines have in order to convert 50-Hz power to 60-Hz power?

SOLUTION: The speed of a synchronous machine is related to its frequency by the equation
\[
\frac{n_m}{P} = \frac{120 f_s}{P}
\]
To make a 50 Hz and a 60 Hz machine have the same mechanical speed so that they can be coupled together, we see that
\[
\frac{120(50 \text{ Hz})}{P_1} = \frac{120(60 \text{ Hz})}{P_2}
\]
\[
\frac{P_2}{P_1} = \frac{6}{5} = \frac{12}{10}
\]
Therefore, a 10-pole synchronous motor must be coupled to a 12-pole synchronous generator to accomplish this frequency conversion.

5-2)
A 480-V 200-kVA 0.8-power-factor-lagging 60-Hz two-pole Y-connected synchronous generator has a synchronous reactance of 0.25 \(\Omega\) and an armature resistance of 0.03 \(\Omega\). At 60 Hz, its friction and windage losses are 6 kW, and its core losses are 4 kW. The field circuit has a dc voltage of 200 V, and the maximum \(I_F\) is 10 A. The resistance of the field circuit is adjustable over the range from 20 to 200 \(\Omega\). The OCC of this generator is shown in Figure P5-1.

(a) How much field current is required to make \(V_T\) equal to 480 V when the generator is running at no load?

(b) What is the internal generated voltage of this machine at rated conditions?

(c) How much field current is required to make \(V_T\) equal to 480 V when the generator is running at rated conditions?

(d) How much power and torque must the generator's prime mover be capable of supplying?
SOLUTION

(a) If the no-load terminal voltage is 480 V, the required field current can be read directly from the open-circuit characteristic. It is 4.55 A.

(b) This generator is Y-connected, so $I_L = I_A$. At rated conditions, the line and phase current in this generator is

$$I_A = I_L = \frac{P}{\sqrt{3} V_L} = \frac{200 \text{ kVA}}{\sqrt{3}(480 \text{ V})} = 240.6 \text{ A} \text{ at an angle of } -36.87^\circ$$

The internal generated voltage of the machine is

$$E_A = V_o + R_A I_A + j X_A I_A$$

$$E_A = 277 \angle 0^\circ + (0.03 \Omega)(240.6 \angle -36.87^\circ \text{ A}) + j(0.25 \Omega)(240.6 \angle -36.87^\circ \text{ A})$$

$$E_A = 322 \angle 7.8^\circ \text{ V}$$

(c) The equivalent open-circuit terminal voltage corresponding to an $E_A$ of 322 volts is

$$V_{oc} = \sqrt{3}(322 \text{ V}) = 558 \text{ V}$$

From the OCC, the required field current is 7.0 A.

(d) The input power to this generator is equal to the output power plus losses. The rated output power is

$$P_{\text{out}} = (200 \text{ kVA})(0.8) = 160 \text{ kW}$$

$$P_{\text{CV}} = 3 I_A^2 R_A = 3(240.6 \text{ A})^2(0.03 \Omega) = 5.2 \text{ kW}$$
\[ P_{\text{F&W}} = 6 \text{ kW} \]
\[ P_{\text{core}} = 4 \text{ kW} \]
\[ P_{\text{stat}} = (\text{assumed 0}) \]
\[ P_{\text{IN}} = P_{\text{OUT}} + P_{\text{CL}} + P_{\text{F&W}} + P_{\text{core}} + P_{\text{stat}} = 175.2 \text{ kW} \]

Therefore the prime mover must be capable of supplying 175 kW. Since the generator is a two-pole 60 Hz machine, it must be turning at 3600 r/min. The required torque is

\[ \tau_{\text{app}} = \frac{P_{\text{IN}}}{\omega_{\text{n}}} = \frac{175.2 \text{ kW}}{(3600 \text{ r/min}) \left( \frac{1 \text{ min}}{60 \text{ s}} \right) \left( \frac{2\pi \text{ rad}}{1 \text{ r}} \right)} = 465 \text{ N \cdot m} \]

5-15)
A 100-MVA 11.5-kV 0.8-PF-lagged 50-Hz two-pole Y-connected synchronous generator has a per-unit synchronous reactance of 0.8 and a per-unit armature resistance of 0.012.

(a) What are its synchronous reactance and armature resistance in ohms?

(b) What is the magnitude of the internal generated voltage \( E_d \) at the rated conditions? What is its torque angle \( \delta \) at these conditions?

(c) Ignoring losses in this generator, what torque must be applied to its shaft by the prime mover at full load?

**SOLUTION** The base phase voltage of this generator is \( V_{d,\text{base}} = 11.500 / \sqrt{3} = 6640 \text{ V} \). Therefore, the base impedance of the generator is

\[ Z_{\text{base}} = \frac{3 \left( \frac{V_{d,\text{base}}}{S_{\text{base}}} \right)^2}{100,000,000 \text{ VA}} = 1.32 \Omega \]

(a) The generator impedances in ohms are:

\[ R_d = \frac{(0.012)(1.32 \Omega)}{} = 0.0158 \Omega \]

\[ X_s = (0.8)(1.32 \Omega) = 1.06 \Omega \]

(b) The rated armature current is

\[ I_d = I_2 = \frac{S}{\sqrt{3}V_f} = \frac{100 \text{ MVA}}{\sqrt{3}(11.5 \text{ kV})} = 5020 \text{ A} \]

The power factor is 0.8 lagging, so \( I_d = 5020 \angle -36.87^\circ \) A. Therefore, the internal generated voltage is

\[ E_d = V_p + R_d I_d + jX_s I_d \]
\[ E_d = 6640 \angle 0^\circ + (0.0158 \Omega)(5020 \angle -36.87^\circ \text{ A}) + j(1.06 \Omega)(5020 \angle -36.87^\circ \text{ A}) \]
\[ E_d = 10,750 \angle 23^\circ \text{ V} \]
Therefore, the magnitude of the internal generated voltage $E_d = 10.750 \text{ V}$, and the torque angle $\delta = 23^\circ$.

(c) Ignoring losses, the input power would equal the output power. Since

$$P_{OUT} = (0.80)(100 \text{ MVA}) = 80 \text{ MW}$$

and

$$n_{mech} = \frac{120 f_d}{P} = \frac{120 (50 \text{ Hz})}{2} = 3000 \text{ r/min}$$

the applied torque would be

$$t_{app} = t_{ind} = \frac{80,000,000 \text{ W}}{(3000 \text{ r/min})(2\pi \text{ rad/r})(1 \text{ min/60 s})} = 255,000 \text{ N \cdot m}$$

UPDATE 10-26-2005

Some versions of the book state PF = 0.85 instead of PF = 0.8. For **PF = 0.85** the following results apply:

Current phase angle $31.79^\circ$

$E_A = 10.513 \text{ kV}, 25.23^\circ$

$P_{OUT} = 85 \text{ MW}$

$t_{app} = t_{ind} = 270.9 \text{ kNm}$