4-6. The flux density distribution over the surface of a two-pole stator of radius \( r \) and length \( l \) is given by

\[
B = B_m \cos(\omega_m t - \alpha)
\]

Prove that the flux density under each pole face is

\[
\phi = 2rlB_m
\]

Solve first specifically for a 2-pole and then for a 4-pole machine and generally for a \( P \)-pole machine

**SOLUTION**

(a) \( P = 2 \): Seen from the rotor the flux density becomes time independent

\[
B = B_m \cos(\alpha), \text{ since the rotor spins with the mechanical angular velocity } \omega_m.
\]

The flux penetrating an infinitesimal small area \( dA = l \cdot r \cdot d\alpha \) is

\[
d\phi = l \cdot r \cdot B_m \cos(\alpha) d\alpha
\]

and the total flux under each pole face is the aerial integral of this \( d\phi \). In a 2-pole machine one pole face (e.g. one north pole) reaches over 180° from \( \alpha = -90^\circ \) to \( \alpha = 90^\circ \) (assuming the peak flux density \( B_m \) at \( \alpha = 0 \)). Therefore, the total flux becomes

\[
\phi = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} rB_m \cos(\alpha) d\alpha = rB_m \sin(\alpha) \bigg|_{-\frac{\pi}{2}}^{\frac{\pi}{2}} = rB_m \left[ \sin\left(\frac{\pi}{2}\right) - \sin\left(-\frac{\pi}{2}\right) \right] = 2rB_m
\]

(b) \( P = 4 \): The procedure of integration is the same as above, except here the flux density distribution of course becomes \( B = B_m \cos(2\alpha) \) and one pole face stretches over only 90° from \( \alpha = -45^\circ \) to \( \alpha = 45^\circ \).

\[
\phi = \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} rB_m \cos(2\alpha) d\alpha = \frac{1}{2} rB_m \sin(2\alpha) \bigg|_{-\frac{\pi}{4}}^{\frac{\pi}{4}} = \frac{1}{2} rB_m \left[ \sin\left(\frac{\pi}{2}\right) - \sin\left(-\frac{\pi}{2}\right) \right] = rB_m
\]

Therefore, the total flux under the pole of a 4-pole machine is half of the flux of a 2-pole machine of same geometric dimensions, provided the flux density is the same.

(c) In general, in a \( P \)-pole machine the flux density distribution is \( B = B_m \cos\left(\frac{P}{2}\alpha\right) \) and one pole stretches over only 360°/\( P \) from \( \alpha = -180^\circ/P \) to \( \alpha = 180^\circ/P \). Therefore, the flux becomes

\[
\phi = \int_{-\frac{\pi}{P}}^{\frac{\pi}{P}} rB_m \cos\left(\frac{P}{2}\alpha\right) d\alpha = \frac{2}{P} rB_m \left[ \sin\left(\frac{\pi}{2}\right) - \sin\left(-\frac{\pi}{2}\right) \right] = \frac{4}{P} rB_m
\]