11.5 Sketch a phasor representation of a balanced three-phase system containing both phase voltages and line voltages if $V_{ab} = 208 \, 60^\circ \text{ V rms}$. Label all phasors and assume an $abc$-phase sequence.

**SOLUTION:**

\[
V_{ab} = 208 \, 60^\circ \text{ V rms} \quad V_{bc} = 208 \, -60^\circ \text{ V rms} \quad V_{ca} = 208 \, 180^\circ \text{ V rms}
\]

\[
V_{an} = \frac{V_{ab}}{\sqrt{3}} \, 30^\circ = 120 \, 30^\circ \text{ V rms}
\]

\[
V_{bn} = 120 \, 90^\circ \text{ V rms} \quad V_{cn} = 120 \, 150^\circ \text{ V rms}
\]
11.32 An *abc*-phase-sequence three-phase balanced wye-connected source supplies power to a balanced delta-connected load. The impedance per phase in the load is $14 + j7 \, \Omega$. If the source voltage for the *a* phase is $V_{an} = 120/80^\circ \, V$ rms and the line impedance is zero, find the phase currents in the wye-connected source.

**SOLUTION:**

$$Z_L = 14 + j7 \, \Omega$$  
$$\frac{Z_L}{3} = \frac{14 + j7}{3} = Z_L$$

$$I_{aA} = \frac{V_{an}}{Z_L}$$

$$I_{aA} = 23/53.4^\circ \, A_{\text{rms}}$$

$$I_{bB} = 23/166.4^\circ \, A_{\text{rms}}$$

$$I_{cC} = 23/173.4^\circ \, A_{\text{rms}}$$
11.44 In a balanced three-phase system, the source has an \( abc \)-phase sequence and is connected in delta. There are two loads connected in parallel. The line connecting the source to the loads has an impedance of \( 0.2 + j0.1 \ \Omega \). Load 1 is connected in wye, and the phase impedance is \( 4 + j2 \ \Omega \). Load 2 is connected in delta, and the phase impedance is \( 12 + j9 \ \Omega \). The current \( I_{AB} \) in the delta load is \( 16/45^\circ \) A rms. Find the phase voltages of the source.

\[
\text{SOLUTION: Per phase } Y
\]

\[
\begin{align*}
Z_{1Y} &= 4 + j2 \ \Omega \\
Z_{2Y} &= Z_2/3 = 4 + j3 \ \Omega \\
I_{AN2} &= I_{AB2} \left( \frac{1}{\sqrt{3}} \angle -30^\circ \right) \\
I_{AN2} &= 27.7/15^\circ \ \text{Ams}
\end{align*}
\]

\[
V_{AN} = I_{AN2} Z_{2Y}
\]

\[
V_{AN} = 1.19 \angle 51.9^\circ \ \text{Vrms} \\
I_{AN1} = \frac{V_{AN}}{Z_{1Y}} = 31.0/25.3^\circ \ \text{Ams}
\]

\[
I_{AN} = I_{AN1} + I_{AN2} = 58.5/20.4^\circ \ \text{Ams}
\]

\[
V_{AN} = I_{AN} (0.2 + j0.1) + V_{AN} = 152/57.5^\circ \ \text{Vrms}
\]

\[
V_{ab} = V_{AN} \left( \sqrt{3} \angle 30^\circ \right) = 263/81.5^\circ \ \text{Vrms}
\]

\[
\begin{align*}
V_{ab} &= 263/81.5^\circ \ \text{Vrms} \\
V_{bc} &= 263/138.5^\circ \ \text{Vrms} \\
V_{ca} &= 263/-158.5^\circ \ \text{Vrms}
\end{align*}
\]
The following loads are served by a balanced three-phase source:

Load 1: 20 kVA at 0.8 pf lagging
Load 2: 4 kVA at 0.8 pf leading
Load 3: 10 kVA at 0.75 pf lagging

The load voltage is 208 V rms at 60 Hz. If the line impedance is negligible, find the power factor at the source.

**SOLUTION:**

\[ |V_{AB}| = 208 \text{ V rms} \quad |V_{AN}| = |V_{AB}| / \sqrt{3} = 120 \text{ V rms} = |V_{An}| \]

Assume \( \theta_{VAN} = 0^\circ \) since \( \frac{V_{line}}{V_{rms}} = 1 \), \( \theta_{Van} = 0^\circ \) also.

**Load 1:** \( S_1 = 20000 / \cos^{-1}(0.8) = 3 \ V_{AN} I_{AN1}^* = 3 (120 \angle 0^\circ) I_{AN1}^* \)

\[ I_{AN1}^* = 55.6 / -36.9^\circ \text{ Arms} \]

**Load 2:** \( S_2 = 4000 / \cos^{-1}(0.8) = 3 \ V_{AN} I_{AN2}^* \)

\[ I_{AN2}^* = 11.1 / +36.9^\circ \text{ Arms} \]

**Load 3:** \( S_3 = 10000 / \cos^{-1}(0.75) = 3 \ V_{AN} I_{AN3}^* \Rightarrow I_{AN3} = 27.8 / -41.4^\circ \text{ Arms} \)

\[ I_{AN} = I_{AN1} + I_{AN2} + I_{AN3} = 86.4 / -31.3^\circ \text{ Arms} \]

\[ \rho_L = \cos(\theta_{VAN} - \theta_{IAN}) = 0.854 \text{ lagging} \]

Since \( V_{AN} = V_{AN} \) (no line impedance)

\[ \rho_S = \rho_L = 0.854 \text{ lagging} \]
A three-phase abc-sequence wye-connected source with \( V_{an} = 220/0^\circ \) V rms supplies power to a wye-connected load that consumes 50 kW of power in each phase at a pf of 0.8 lagging. Three capacitors are found that each have an impedance of \(-j2.0 \ \Omega\), and they are connected in parallel with the previous load in a wye configuration. Determine the power factor of the combined load as seen by the source.

**SOLUTION:**

\[ P_{1\phi} = 50 \text{ kW} \quad \text{pf} = 0.8 \text{ lagging} \]

\[ P_{1\phi} = |V_{an}|I_{\text{an}}|/\text{pf} \Rightarrow |I_{\text{an}}| = \frac{50,000}{220} = 224 \text{ Arms} \]

\[ \Theta_{V_{an}} - \Theta_{I_{\text{an}}} = \cot^{-1} \text{(pf)} = 36.9^\circ \quad \Theta_{I_{\text{an}}} = -36.9^\circ \]

\[ I_{\text{an}} = 224/\angle36.9^\circ \text{ Arms} \quad \Theta_Z = \Theta_S = 36.9^\circ \]

\[ Q_{1\phi} = |S_{1\phi}| \sin (\Theta_S) \quad |S_{1\phi}| = P_{1\phi}/\text{pf} = 62.5 \text{ kVA} \]

\[ Q_{1\phi} = 57.5 \text{ kVAR} \]

\[ Q_L = |V_{an}|^2/X_C = \frac{220^2}{2} = 224 \text{ kVAR} \]

\[ Q_{\text{new}} = Q_{1\phi} + Q_L = 153.3 \text{ kVAR} \]

\[ S_{\text{new}} = P_{1\phi} + jQ_{\text{new}} = 50 + j153.3 \text{ kVA} \]

\[ \Theta_{S_{\text{new}}} = \tan^{-1} \left( \frac{Q_{\text{new}}}{P_{1\phi}} \right) = 14.9^\circ \quad \text{pf}_{\text{new}} = \cos (\Theta_{S_{\text{new}}}) \]

\[ \text{pf}_{\text{new}} = 0.966 \text{ lagging} \]