Robust Fault Detection and Isolation Using Robust ℓ_1 Estimation

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Abstract

This paper considers the application of robust ℓ_1 estimation to robust fault detection and isolation. This is accomplished by developing a series, or bank, of robust estimators (full-order observers), each of which is designed such that the residual will be sensitive to a certain fault (or faults) while insensitive to the remaining faults. Robustness is incorporated by assuring that the residual remains insensitive to exogenous disturbances as well as modeling uncertainty. Mixed structured singular value and ℓ_1 theories are used to develop the appropriate threshold logic to evaluate the outputs of the estimators used for determining the occurrence and location of a fault. A real-coded genetic algorithm is used to obtain the estimator gain matrices. This approach to FDI is successfully demonstrated using a linearized model of a jet engine.

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Nomenclature

$\mathcal{R}, \mathcal{C}, \mathcal{Z}^+$	real numbers, complex numbers, nonnegative integers
$\mathcal{R}^{m\times n}, \mathcal{C}^{m\times n}$	$m \times n$ real matrices, complex matrices
$\mathcal{D}^n, \mathcal{N}^n, \mathcal{P}^n$	$n \times n$ real diagonal, nonnegative definite, positive definite matrices
$0,\ I$	zero matrix, identity matrix
tr	trace
$M_2 > M_1$	$M_2 - M_1$ positive definite
$M_2 \geq M_1$	$M_2 - M_1$ nonnegative definite
$\dim(M)$	dimension of M
$\Vert z(\cdot)\Vert_{\infty,2}$	$ess\sup_{t\geq 0} \vert\vert z(t)\vert\vert_2$
$\Vert z(\cdot)\Vert_{(\infty,2),[N_0,N]}$	$ess\sup_{t\in[N_0T,NT]} \vert\vert z(t)\vert\vert_2$
$\Vert H_{zw}\Vert_1$	$\sup_{w(\cdot)\in\ell_\infty} \frac{\vert\vert z\vert\vert_{\infty,2}}{\vert\vert w\vert\vert_{\infty,2}}$
z_{ij}	(i,j) element of matrix Z
$Vec(Z),\ Z \in \mathcal{R}^{m\times n}$	$[z_{11},\ \ldots,\ z_{m1},\ z_{12},\ \ldots,\ z_{m2},\ \ldots,\ z_{mn}]^T$
$diag(Z),\ Z \in \mathcal{D}^n$	$[z_{11},\ z_{22},\ \ldots,\ z_{nn}]^T$

Introduction

In modern systems such as aircraft and spacecraft, there is an increasing demand for reliability and safety. For example, a jet engine is very critical for an aircraft and if faults occur, the consequences can be extremely serious.[1] Many dynamic systems are complex technical systems that involve extensive use of multiple sensors, actuators and other system components, any one of which could fail or deteriorate. Hence, health monitoring and supervision of these systems is essential for the improvement of reliability, safety and dependability of operations. This entails continuously checking a physical system for faults and taking appropriate actions to maintain the operation in such situations. In particular, the objective is to detect and isolate failures or anomalies in the sensors, actuators and components.

One of the primary approaches to model-based, fault detection and isolation uses state or output estimators.[2–8] Detection of a fault is achieved by comparing the actual behavior of the plant to that expected on the basis of the model; deviations are indications of a fault (or disturbances, noise or modeling errors).[9] Fault isolation can be achieved by dedicating an estimator such that the residual is sensitive to only one particular fault. In particular, referring to Figure 1, a bank of estimators is used to generate residuals $r(t)$. These residuals are then analyzed by some appropriate logic (e.g., logic based on thresholds or fuzzy logic) which infers whether faults have occurred (fault detection) and where they have occurred (fault isolation).

In many approaches to the FDI problem the robustness aspect is commonly introduced in relation to the fault detection.[10] The estimators shown in Figure 1 may be designed in a variety of ways, for example by using Kalman filter theory (i.e., H_2 optimal estimation),[11–13] H_∞ theory,[14–16] or ℓ_1 theory.[17–19] Whichever method is used for designing the estimator, it will use an idealized mathematical description of the underlying plant. In practice this model of the plant is never totally accurate, which can degrade the quality of the residuals produced by the estimators. The errors in the plant model may be either parametric or unstructured (e.g., unmodeled dynamics). To reduce the degradation in the quality of the residuals upon which the FDI process is based, and hence to reduce the false alarm rate, it is imperative that the plant uncertainty be explicitly taken into account in the design of the estimators.

Until recent work,[11,14,18,20] the relatively nonconservative mixed structured singular value (MSSV) techniques[21–24] had not been applied to robust estimation, although more conservative techniques, based on the small gain theorem or fixed quadratic Lyapunov functions, had been used.[15,16,25–27] Conservatism in robustness theory involves how the theory actually models the uncertainty. For example, even if the uncertainty is real and parametric, the small gain theorem assumes that the uncertainty is complex and unstructured. Likewise, fixed quadratic Lyapunov function theory assumes that the uncertainty is arbitrarily time-varying. MSSV theory, which considers both parametric uncertainty and unmodelled dynamics, allows real parametric uncertainty to be treated as slowly time-varying, real parametric uncertainty, which is a much less conservative model. The reduced conservatism allows the estimators to be used for more accurate fault detection. Specifically, the fixed thresholds are smaller, allowing the detection of smaller faults. With more conservative theories the thresholds are larger, causing some smaller faults to go undetected. Although the example in this paper focuses exclusively on sensor faults the theory is developed to include actuator faults as well.

This paper considers the application of robust ℓ_1 estimation to fault robust fault detection and isolation. This is accomplished by developing a series, or bank, of robust estimators (full-order observers), each of which is designed such that the residual will be sensitive to a certain fault (or faults) while insensitive to the remaining faults. Robustness is incorporated by assuring that the residual remains insensitive to exogenous disturbances as well as modelling uncertainty. Mixed structured singular value and ℓ_1 theories are used to develop the appropriate threshold logic to evaluate the outputs of the estimators used for determining the occurrence and location of a fault. A real-coded genetic algorithm is used to obtain the estimator gain matrices. The effectiveness of this robust FDI technique is illustrated as it is applied to a discrete-time, linear uncertain model of an advanced afterburning turbofan engine.

The organization of this paper is as follows. Section 2 presents the formulation of the closed-loop uncertain system to which estimation will be applied. The application of robust ℓ_1 estimation to robust fault detection and isolation is presented in Section 3. Section 4 discusses results of an illustrative example of a jet engine, while Section 5 gives concluding remarks.

Robust ℓ_1 Estimation

Consider a discrete-time, linear uncertain dynamic system

$$x(k+1) = (A + \Delta A)x(k) + Bu(k) + D_{\infty,1}w_\infty(k), \quad k \in \mathcal{Z}^+ \tag{1}$$
$$y(k) = (C + \Delta C)x(k) + Du(k) + D_{\infty,2}w_\infty(k), \tag{2}$$
$$z(k) = E_\infty x(k), \tag{3}$$

where $x \in \mathcal{R}^n$ is the state vector, $u \in \mathcal{R}^d$ is the control input, $y \in \mathcal{R}^p$ denotes the plant measurements, $z \in \mathcal{R}^q$ is the performance output to be estimated, and $w_\infty \in \mathcal{R}^{d_\infty}$ denotes an ℓ_∞ disturbance signal satisfying $\|w_\infty(\cdot)\|_{\infty,2} \leq 1$. The uncertainties ΔA, ΔB and ΔC satisfy

$$\Delta A \in \mathcal{U}_A \triangleq \{\Delta A \in \mathcal{R}^{n\times n} : \Delta A = -H_A F_A G_A, \;\; F_A \in \mathcal{F}_A\}, \tag{4}$$
$$\Delta C \in \mathcal{U}_C \triangleq \{\Delta C \in \mathcal{R}^{p\times n} : \Delta C = -H_C F_C G_C, \;\; F_C \in \mathcal{F}_C\}, \tag{5}$$

where

$$\mathcal{F}_A \triangleq \{F_A \in \mathcal{D}^r : M_{A,1} \leq F_A \leq M_{A,2}\}, \tag{6}$$
$$\mathcal{F}_C \triangleq \{F_C \in \mathcal{D}^t : M_{C,1} \leq F_C \leq M_{C,2}\}, \tag{7}$$

with $M_{A,1},\ M_{A,2} \in \mathcal{D}^r,\ M_{C,1},\ M_{C,2} \in \mathcal{D}^t,\ M_{A,2} - M_{A,1} \geq 0$, and $M_{C,2} - M_{C,1} \geq 0$.

It is desired to design a full-order observer of the form

$$x_e(k+1|k) = A_e x_e(k|k-1) + (B - WD)u(k) + W[y(k) - Cx_e(k|k-1)] \tag{8}$$

to estimate the state vector x, where $W \in \mathcal{R}^{n\times p}$ and $A_e \in \mathcal{R}^{n\times n}$ are the parameters to be determined.

The state estimation error is defined as

$$e(k) \triangleq x(k) - x_e(k|k-1), \tag{9}$$

which using (1), (2), and (8) can be shown to obey the evolution equation

$$e(k+1) = (A_e - WC)e(k) + (A - A_e + \Delta A - W\Delta C)x(k) + (D_{\infty,1} - WD_{\infty,2})w_\infty(k). \tag{10}$$

Now define the error output $\tilde{z} \in \mathcal{R}^{q_p}$ as $\tilde{z}(k) \triangleq E_\infty e(k)$. Then augmenting (1) with (10) yields

$$\tilde{x}(k+1) = (\tilde{A} + \Delta\tilde{A})\tilde{x}(k) + \tilde{D}_1 w_\infty(k), \tag{11}$$

$$\tilde{z}(k) = \tilde{E}\tilde{x}(k), \tag{12}$$

where

$$\tilde{x}(k) = \begin{bmatrix} x(k) \\ e(k) \end{bmatrix}, \quad \tilde{A} = \begin{bmatrix} A & 0 \\ A - A_e & A_e - WC \end{bmatrix},$$

$$\tilde{D}_1 = \begin{bmatrix} D_{\infty,1} \\ D_{\infty,1} - WD_{\infty,2} \end{bmatrix}, \; \tilde{E} = \begin{bmatrix} 0 & E \end{bmatrix}. \tag{13}$$

Furthermore, $\Delta\tilde{A}$ satisfies

$$\Delta\tilde{A} \in \tilde{\mathcal{U}}_A \triangleq \{\Delta\tilde{A} \in \mathcal{R}^{2n\times 2n} : \Delta\tilde{A} = -\tilde{H}_A\tilde{F}_A\tilde{G}_A, \; \tilde{F}_A \in \tilde{\mathcal{F}}_A\}, \tag{14}$$

$$\tilde{\mathcal{F}}_A \triangleq \{\tilde{F}_A \in \mathcal{D}^{r+t} : M_1 \leq \tilde{F}_A \leq M_2\}, \tag{15}$$

where

$$\tilde{F}_A = \begin{bmatrix} F_A & 0 \\ 0 & F_C \end{bmatrix}, \quad \tilde{H}_A = \begin{bmatrix} H_A & 0 \\ H_A & -WH_C \end{bmatrix}, \quad \tilde{G}_A = \begin{bmatrix} G_A & 0 \\ G_C & 0 \end{bmatrix}, \tag{16}$$

and

$$M_1 = \mathrm{diag}(M_{A,1}, M_{C,1}), \quad M_2 = \mathrm{diag}(M_{A,2}, M_{C,2}). \tag{17}$$

The robust ℓ_1 estimation problem is to find the estimator parameters A_e and W such that the combined system (8), (11)-(12) is asymptotically stable and the cost functional

$$\mathcal{J}(W) = \|H_{zw}\|_1^2, \tag{18}$$

is minimized, where H_{zw} is the the convolution operator from the disturbance $w_\infty(\cdot)$ to the ℓ_∞ performance variable $\tilde{z}(\cdot)$.

As shown in,[28] direct minimization of the ℓ_1 norm can lead to irrational estimators. However, Haddad et. al.[29] shows it is possible to characterize an upper bound on the ℓ_1 performance. For some uncertainty set $\mathcal{U} \subset \mathcal{R}^{n\times n}$, $\Delta A \in \mathcal{U}$, $x \in \mathcal{R}^n$, $z \in \mathcal{R}^q$ $w_\infty(\cdot) \in \mathcal{R}^{d_\infty}$, the ℓ_1 performance bound as a function of $\Delta\tilde{A}$ is given in the following proposition

Proposition 1 *Let $\alpha > 1$ and assume there exists a positive-definite matrix $Q_{\Delta A}$ satisfying*

$$Q_{\Delta\tilde{A}} = \alpha(\tilde{A} + \Delta\tilde{A})Q_{\Delta\tilde{A}}(\tilde{A} + \Delta\tilde{A})^T + \frac{\alpha}{\alpha - 1}V_\infty, \tag{19}$$

where $V_\infty \triangleq \tilde{D}_1\tilde{D}_1^T$. Then $\tilde{A} + \Delta\tilde{A}$ is asymptotically stable. Furthermore, the ℓ_1 norm of the convolution operator H_{zw} from disturbances $w(\cdot)$ to the performance variable $\tilde{z}(\cdot)$ satisfies the bound

$$\|H_{zw}\|_1^2 \leq \sup_{\Delta\tilde{A}\in\mathcal{U}} [tr(\tilde{E}Q_{\Delta\tilde{A}}\tilde{E}^T)^q]^{\frac{1}{q}}, \;\; \Delta\tilde{A} \in \mathcal{U}. \tag{20}$$

If, in addition, α is chosen such that $\sqrt{\alpha}(\tilde{A}+\Delta\tilde{A})$ is asymptotically stable, then the existence of a positive-definite solution $Q_{\Delta\tilde{A}}$ is guaranteed.

Proof. Follows from Lemma in.[18]

Remark 1 *Minimization of the upper bound is a more appropriate approach than some conventional methods. Typical linear programming methods[30] that seek to directly minimize the ℓ_1 norm do not allow a fixed architecture estimator or controller design. These methods normally result in very high order estimators or controllers, which are not practical for implementation.*

In order to obtain an upper bound on the ℓ_1 performance $\|H_{zw}\|_1^2$ over the entire uncertainty set $\mathcal{U}$, a multiplier framework will be used to bound $Q_{\Delta\tilde{A}}$ for all $\Delta A \in \mathcal{U}$ where

$$\mathcal{U} \triangleq \{\Delta\tilde{A} \in \mathcal{R}^{2n\times 2n} : \Delta\tilde{A} = -H_0FG_0, \;\; F \in \mathcal{F}\}. \tag{21}$$

Let $G(z) \in \mathcal{C}^{q\times d_\infty}$ be the transfer function representation of the system described in (11) and (12). The Popov-Tsypkin multiplier[11,14,18,24] has the transfer function form

$$M(z) = H + N\frac{z-1}{z}, \tag{22}$$

where $H \in \mathcal{D}^m$, $N \in \mathcal{D}^m$ $(m = r + t)$ with $H > 0$ and $N \geq 0$. Let A_a denote the state matrix of the augmented system $M(z)G(z)$. Then, the uncertain system for robust analysis is given by[31]

$$x_a(k+1) = (A_a + \Delta A_a)x_a(k) + D_{a,\infty}w_\infty(k), \tag{23}$$
$$z(k) = E_a x_a(k), \tag{24}$$

where $x_a(k) = [x_m^T(k)x^T(k)]^T$, $x_m(k) \in \mathcal{R}^m$ denotes the states of the multiplier,

$$A_a = \begin{bmatrix} 0 & 0 \\ H_0 & \tilde{A} \end{bmatrix}, \quad D_{a,\infty} = \begin{bmatrix} 0 \\ D_\infty \end{bmatrix}, \quad E_a = \begin{bmatrix} 0 & E \end{bmatrix}, \tag{25}$$

and

$$\Delta A_a \in \mathcal{U}_a \triangleq \{\Delta A_a \in \mathcal{R}^{m+n} : \Delta A_a = -H_a F G_a, \quad F \in \mathcal{F}\}, \tag{26}$$

where

$$H_a = \begin{bmatrix} 0 \\ H_0 \end{bmatrix}, \quad G_a = \begin{bmatrix} 0 & G_0 \end{bmatrix}. \tag{27}$$

Note that the uncertainty set $\mathcal{U}$ in (21) is a subset of $\mathcal{U}_a$. The next theorem provides an upper bound for the ℓ_1 performance for all $\Delta A_a \in \mathcal{U}_a$.

Theorem 1 *Let $\alpha > 1$, $q \geq 1$. Suppose there exists $H \in \mathcal{P}^n$, $N \in \mathcal{N}^n$ and $Q_a \geq 0$ such that $2H(M_2 - M_1)^{-1} - G_a Q_a G_a^T > 0$, and Q_a satisfies the algebraic Riccati equation*

$$\begin{aligned} Q_a = {}& \alpha(A_a - H_a M_1 G_a)Q_a(A_a - H_a M_1 G_a)^T + [\sqrt{\alpha}(A_a - H_a M_1 G_a)Q_a C_a^T - \sqrt{\alpha}B_a(H+N) \\ & + S_a N][2H(M_2 - M_1)^{-1} - G_a Q_a G_a^T]^{-1}[\sqrt{\alpha}(A_a - H_a M_1 G_a)Q_a C_a^T - \sqrt{\alpha}B_a(H+N) \\ & + S_a N]^T + \frac{\alpha}{\alpha - 1}V_{a,\infty}, \end{aligned} \tag{28}$$

where $V_{a,\infty} \triangleq D_{a,\infty}D_{a,\infty}^T$ and $S_a \triangleq \begin{bmatrix} I \\ 0 \end{bmatrix}$, where $\dim(S_a) = \dim(H_a)$.

Then,

$$\{(A_a + \Delta A_a), [\frac{\alpha}{\alpha - 1}V_{a,\infty}]^{\frac{1}{2}}\} \text{ is stabilizable, } \Delta A \in \mathcal{U}, \tag{29}$$

if and only if $(A_a + \Delta A_a)$ is asymptotically stable for each $\Delta A_a \in \mathcal{U}_a$ and in this case, the ℓ_1 performance $\|H_{zw}\|_1^2$ is bounded as

$$\|H_{zw}\|_1^2 \leq [tr(E_a Q_a E_a^T)^q]^{\frac{1}{q}}, \quad \Delta A_a \in \mathcal{U}_a. \tag{30}$$

Proof. The proof can be completed in the same manner as proof to results in.[23]

Now consider the fault driven system

$$x(k+1) = (A+\Delta A)x(k) + Bu(k) + D_{\infty,1}w_\infty(k) + R_a f_a(k), \tag{31}$$
$$y(k) = (C+\Delta C)x(k) + D_p u(k) + D_{\infty,2}w_\infty(k) + R_s f_s(k), \tag{32}$$
$$z(k) = E_\infty x(k), \tag{33}$$

where $f_a \in \mathcal{R}^{r_a}$ and $f_s \in \mathcal{R}^{r_s}$ are the actuator and sensor fault vectors, respectively. The fault distribution matrices R_a and R_s are assumed to be known. Defining $f \triangleq [f_a^T \; f_s^T]^T$ yields the modified system

$$x(k+1) = (A+\Delta A)x(k) + Bu(k) + D_{\infty,1}w_\infty(k) + R_1 f(k), \tag{34}$$
$$y(k) = (C+\Delta C)x(k) + D_{\infty,2}w_\infty(k) + R_2 f(k), \tag{35}$$
$$z(k) = E_\infty x(k), \tag{36}$$

where

$$R_1 = \begin{bmatrix} R_a & 0 \end{bmatrix}, \quad R_2 = \begin{bmatrix} 0 & R_s \end{bmatrix}. \tag{37}$$

Equations (34)-(35) can be written in the expanded forms

$$x(k+1) = (A+\Delta A)x(k) + Bu(k) + D_{\infty,1}w_\infty(k) + R_{1,1}f_1(k) + \ldots + R_{1,g}f_g(k), \tag{38}$$
$$y(k) = (C+\Delta C)x(k) + D_{\infty,2}w_\infty(k) + R_{2,1}f_1(k) + \ldots + R_{2,g}f_g(k), \tag{39}$$

where $R_{1,i}$ (respectively, $R_{2,i}$) denotes the i^{th} column of the matrix R_1 (respectively, R_2). Let $g \triangleq r_a + r_s$. For $i \in \{1, 2, \ldots, g\}$ the term $f_i(k)$ represents the i^{th} individual fault of $f(k)$ and $R_{1,i}$ (respectively, $R_{2,i}$) represents its directional characteristics. Assume that $f_i(k)$ is the "target fault," i.e., the fault that it is desired to detect. Without loss of generality, the vector of "nuisance faults", representing the faults that are *not* to be detected (by the robust fault detection filter), is given by $\bar{f}_i \triangleq [f_1(k) \cdots f_{i-1}(k) \; f_{i+1}(k) \cdots f_g(k)]$. Hence, (38)-(39) can be replaced by

$$x(k+1) = (A+\Delta A)x(k) + Bu(k) + D_{\infty,1}w_\infty(k) + R_{1,i}f_i(k) + \bar{R}_{1,i}\bar{f}_i(k), \tag{40}$$
$$y(k) = (C+\Delta C)x(k) + D_{\infty,2}w_\infty(k) + R_{2,i}f_i(k) + \bar{R}_{2,i}\bar{f}_i(k). \tag{41}$$

Define $\tilde{w} \triangleq [w^T \ \bar{f}_i^T]^T$. Then, (40) and (41) can be written as a set of systems

$$\Sigma_i \begin{cases} x(k+1) = (A + \Delta A)x(k) + Bu(k) + D_1\tilde{w}(k) + R_{1,i}f_i(k), \\ \quad y(k) = Cx(k) + D_2\tilde{w}(k) + R_{2,i}f_i(k), \\ \quad z(k) = E_\infty x(k) \end{cases} \tag{42}$$

where

$$D_1 = \begin{bmatrix} D_{\infty,1} & \bar{R}_{1,i} \end{bmatrix}, \ \ D_2 = \begin{bmatrix} D_{\infty,2} & \bar{R}_{2,i} \end{bmatrix}. \tag{43}$$

It is desired to design a bank of full-order observers (corresponding to each faulty system) described in (8) to estimate the performance output $E_\infty x(k)$. As previously stated, $A_e \in \mathcal{R}^{n\times n}$ and $W \in \mathcal{R}^{n\times p}$ are the parameters to be determined. (Typically, one chooses $E_\infty = C$ such that z corresponds to the noise and fault free output associated with the measurement y.) Detection of a fault is achieved by comparing the actual behavior of the plant to the the output of the estimators; deviations are indications of a fault (or disturbances, noise or modeling errors).

Let the residual error be defined as

$$r(k) \triangleq P[y(k) - Cx_e(k|k-1)) - Du(k)], \tag{44}$$

where the $g \times p$ gain matrix P[32] is chosen such that r has a fixed direction when responding to the target fault. Fault isolation can be achieved by designing an estimator such that estimation error (i.e. the residual) is sensitive to only one particular fault. Specifically, each is designed to be sensitive to a particular fault and insensitive to the remaining faults. In addition, these estimators are made robust against exogenous disturbances and modeling uncertainties. Referring to Figure 1, the bank of estimators is used to generate residuals $r(k)$. These residuals are then analyzed by some appropriate logic (e.g., logic based on thresholds) which infers whether faults have occurred (fault detection) and where they have occurred (fault isolation).

Using (42), the state estimation error in (9) can be shown to obey the evolution equation

$$\begin{aligned} e(k+1) =& (A_e - WC)e(k) + (A + \Delta A - W\Delta C - A_e)x(k) \\ &+ (D_1 - WD_2)w_\infty(k) + (R_{1,i} - WR_{2,i})f_i(k). \end{aligned} \tag{45}$$

Augmenting (42) with (45) yields

$$\tilde{x}(k+1) = (\tilde{A} + \Delta\tilde{A})\tilde{x}(k) + \tilde{D}_1 w_\infty(k) + \tilde{R}_1 f_i(k), \tag{46}$$

$$\tilde{z}(k) = \tilde{E}\tilde{x}(k), \tag{47}$$

where

$$\tilde{x}(k) = \begin{bmatrix} x(k) \\ e(k) \end{bmatrix}, \quad \tilde{A} = \begin{bmatrix} A & 0 \\ A - A_e & A_e - WC \end{bmatrix},$$
$$\tilde{D}_1 = \begin{bmatrix} D_1 \\ D_1 - WD_2 \end{bmatrix}, \quad \tilde{R}_1 = \begin{bmatrix} R_{1,i} \\ R_{1,i} - WR_{2,i} \end{bmatrix}, \quad \tilde{E} = \begin{bmatrix} 0 & E_\infty \end{bmatrix}. \tag{48}$$

Let J_{rw} represent the ℓ_1 norm of the system operator from the disturbance vector $\tilde{w}$ to the residual r and let J_{rf} represent the ℓ_1 norm of the system operator from the target fault f_i to the residual r. Following previous derivations, it is possible to characterize upper bounds $\mathcal{J}_{rw}$ and $\mathcal{J}_{rf}$ such that

$$J_{rw} = \|H_{rw}\|_1^2 \leq \mathcal{J}_{rw}, \tag{49}$$

$$J_{rf} = \|H_{rf}\|_1^2 \leq \mathcal{J}_{rf}. \tag{50}$$

Using multiplier theory, the uncertain system is given by

$$x_a(k+1) = (A_a + \Delta A_a)x_a(k) + D_{a,w}\tilde{w}(k) + D_{a,f} f_i(k), \tag{51}$$

$$\tilde{z}(k) = E_a x_a(k), \tag{52}$$

where $x_a(k) = [x_m^T(k)\ \tilde{x}^T(k)]^T$ and $x_m(k) \in \mathcal{R}^m$ is as previously described. The ℓ_1 performance functions then have the bounds,

$$\mathcal{J}_{rw}(A_e, W, P, H, N) = [\mathrm{tr}(E_a Q_{a,w} E_a^T)^q]^{\frac{1}{q}}, \tag{53}$$

$$\mathcal{J}_{rf}(A_e, W, P, H, N) = [\mathrm{tr}(E_a Q_{a,f} E_a^T)^q]^{\frac{1}{q}}, \quad \Delta A_a \in \mathcal{U}_a, \tag{54}$$

where $Q_{a,w}$ and $Q_{a,f}$ satisfy the algebraic Riccati equations

$$
\begin{aligned}
Q_{a,w} &= \alpha(A_a - H_aM_1G_a)Q_{a,w}(A_a - H_aM_1G_a)^T + [\sqrt{\alpha}(A_a - H_aM_1G_a)Q_{a,w}C_a^T \\
&\quad - \sqrt{\alpha}H_a(H+N) + S_aN][2H(M_2 - M_1)^{-1} - G_aQ_{a,w}G_a^T]^{-1}[\sqrt{\alpha}(A_a - H_aM_1G_a)Q_{a,w}C_a^T \\
&\quad - \sqrt{\alpha}H_a(H+N) + S_aN]^T + \frac{\alpha}{\alpha-1}V_{a,w}, \qquad (55)\\
Q_{a,f} &= \alpha(A_a - H_aM_1G_a)Q_{a,f}(A_a - H_aM_1G_a)^T + [\sqrt{\alpha}(A_a - H_aM_1G_a)Q_{a,f}C_a^T \\
&\quad - \sqrt{\alpha}H_a(H+N) + S_aN][2H(M_2 - M_1)^{-1} - G_aQ_{a,f}G_a^T]^{-1}[\sqrt{\alpha}(A_a - H_aM_1G_a)Q_{a,f}C_a^T \\
&\quad - \sqrt{\alpha}H_a(H+N) + S_aN]^T + \frac{\alpha}{\alpha-1}V_{a,f}, \qquad (56)
\end{aligned}
$$

where $V_{a,w} \triangleq D_{a,w}D_{a,w}^T$, $V_{a,f} \triangleq D_{a,f}D_{a,f}^T$, and $S_a \triangleq \begin{bmatrix} I \\ 0 \end{bmatrix}$, with $\dim(S_a) = \dim(H_a)$.

Robust FDI filter design may be approached by choosing A_e, W and P such that $\mathcal{J}_{rw}$ is small and $\mathcal{J}_{rf}$ is large.[1] A minimization problem that expresses this objective is

$$
\min_{A_e,W,P} \; \mathcal{J} = \beta\mathcal{J}_{rw} + (1-\beta)\frac{1}{\mathcal{J}_{rf}} + \gamma\frac{\mathcal{J}_{rw}}{\mathcal{J}_{rf}}, \qquad (57)
$$

where $\beta \in [0,1]$ and $\gamma > 0$ are arbitrarily chosen weighting scalars. With an enforced stability constraint, this optimization problem can be solved using a real-coded genetic algorithm, as discussed in the next section.

Now consider the set of uncertain, discrete-time systems

$$
\begin{aligned}
x(k+1) &= (A + \Delta A)x(k) + D_1w_\infty(k) + R_{1,i}f_i(k), \qquad (58)\\
y(k) &= Cx(k) + D_2w_\infty(k) + R_{2,i}f_i(k), \qquad (59)\\
z(k) &= E_\infty x(k), \qquad (60)
\end{aligned}
$$

where x, y, w, and f_i are as previously discussed. **The robust fault detection problem** is to generate a set robust residual signals $r(k)$ that satisfy

$$
\begin{aligned}
\|r(k)\|_p &\le J_{th} \;\text{ if }\; f_i(k) = 0, \qquad (61)\\
\|r(k)\|_p &> J_{th} \;\text{ if }\; f_i(k) \ne 0, \qquad (62)
\end{aligned}
$$

where $\|\cdot\|_p$ denotes the p norm of a Lebesgue signal and J_{th} is the i^{th} threshold value. If

[1] It would be more desirable to make a *lower* bound on $\mathcal{J}_{rf}$ large. Unfortunately, lower bounds are usually much more difficult to work with computationally than upper bounds.

the filters (8) are applied to (58)-(59) and E_∞ is chosen as C, (44) can be written as

$$r(k) = Pz(k) + PD_2 w_\infty(k) + PR_{2,i} f_i(k). \tag{63}$$

As derived in,[18] if $f_i(k) = 0$ (63) satisfies the norm inequality

$$\begin{aligned}\|r\|^2_{(\infty,2),[N_0,N]} \leq &\{[\mathrm{tr}(PE_aQ_{a,w}E_a^TP^T)^q]^{\frac{1}{q}} + 2\sigma_{\max}(PD_2)[\mathrm{tr}(PE_aQ_{a,w}E_a^TP^T)^q]^{\frac{1}{2q}} \\ &+ \sigma^2_{\max}(PD_2)\}\|\tilde{w}\|^2_{(\infty,2),[N_0,N]},\end{aligned} \tag{64}$$

where $Q_{a,w}$ is previously defined. The threshold can be chosen as

$$\begin{aligned}J_{th} \triangleq &\{[\mathrm{tr}(PE_aQ_{a,w}E_a^TP^T)^q]^{\frac{1}{q}} + 2\sigma_{\max}(PD_2)[\mathrm{tr}(PE_aQ_{a,w}E_a^TP^T)^q]^{\frac{1}{2q}} \\ &+ \sigma^2_{\max}(PD_2)\}\|\tilde{w}\|^2_{(\infty,2),[N_0,N]}.\end{aligned} \tag{65}$$

Robust fault detection can be accomplished by comparing $\|r\|_{(\infty,2),[N_0,N]}$ with J_{th}. A fault occurs if $\|r\|_{(\infty,2),[N_0,N]} > J_{th}$, i.e.,

$$\|r\|_{(\infty,2),[N_0,N]} > J_{th} \Rightarrow \text{a fault occurred.} \tag{66}$$

Optimization Using a Real-Coded Genetic Algorithm

As previously discussed, the design of robust FDI estimators is formulated as an optimization problem. A real-coded genetic algorithm (RCGA) is used to search for a solution. Genetic algorithms (GAs) can efficiently search in complex and possibly discontinuous solution spaces without problem reformulation or evaluation of each solution candidate. They offer the following additional advantages over traditional methods: (1) information about derivatives, Hessians or step sizes is not required, (2) a population of points in the solution space is searched in parallel rather than point by point, and (3) a number of potential solutions to a given problem can be provided. GAs have been proven to provide efficiency (i.e. faster computation times and smaller storage) and flexibility (i.e. adaptation to a range of complex problems) in comparison to traditional methods of optimization. The use of RCGAs, where operations are performed with real numbers, rather than binary GAs, where binary digits are used, proves to be more advantageous. Because no coding or decoding of binary numbers is necessary a subsequent decrease in computational time and storage size is achieved.

An RCGA begins with an arbitrarily chosen initial population within the search region. The algorithm then follows three general operations: (1) *selection*, (2) *recombination*, and (3) *mutation*.[33–36] The flowchart for a single population RCGA is shown in Figure 2.

Selection. A common selection process in RCGAs is conducted using stochastic universal sampling.[35] Individuals of a population are mapped to a line segment, such that each individual's segment is equal in size to its normalized fitness value. Then, N equally spaced pointers are placed along the line segment, where N is the number of individuals to be selected. The position of the first pointer is determined by a randomly generated number $p \in [0, \frac{1}{N}]$ where $\frac{1}{N}$ is the spacing between pointers. This method of selection is analogous to roulette wheel selection[34] and is illustrated in Figure 3 for a population of 8 individuals, n_i, with $N = 4$. From this example it can be seen that individuals n_2, n_3, n_5 and n_7 are chosen.

Recombination. In an RCGA recombination is parallel to crossover in a binary GA. It is the process by which new chromosomes are produced from existing ones and involves the exchange of the individuals' numeric values (genes).[35,36] Let p_1 and p_2 represent two individuals (parents) who are to reproduce. The offspring p_1' and p_2' are produced as a linear combination of the parents:

$$p_1' = \alpha p_1 + (1 - \alpha) p_2, \tag{67}$$

$$p_2' = (1 - \alpha) p_1 + \alpha p_2, \tag{68}$$

where $\alpha \in [0, 1]$ is a recombination parameter.

Mutation. The mutation process was originally developed for binary representation. However, other methods have been developed to allow gene modification in an RCGA. The mutation operation randomly alters one or more genes of a selected chromosome. More specifically, randomly generated values are added to the genes with low probability. The probability of mutation is inversely proportional to the number of variables (dimensions). The more dimensions an individual has, the smaller the mutation probability.[35] An effective mutation operator, which produces small step sizes with a high probability and large step sizes with a low probability, is defined as

$$Gen_i^{mut} = Gen_i + s_i r_i a_i, \quad i \in \{1, 2, \ldots, m\} \text{ uniform at random} \tag{69}$$

$$s_i \in \{-1, +1\}, \text{ uniform at random} \tag{70}$$

$$a_i = 2^{-uk}, \ u \in [0, 1], \text{ uniform at random} \tag{71}$$

where s, r, and a are direction, mutation range, and relative step size, respectively, and m is the number of genes in the chromosome. The mutation range is defined in terms of the domain of the genes, and the step size is defined in terms of the mutation precision k.

For the robust ℓ_1 optimization problem the chromosome is constructed by formulating matrices A_e, W, P, H and N into a single vector Θ such that

$$\Theta = \left[\begin{array}{ccccc} Vec(A_e)^T & Vec(W)^T & Vec(P)^T & diag(H)^T & diag(N)^T \end{array} \right]. \tag{72}$$

The search region is then defined by establishing upper and lower limits $\overline{\Theta}$ and $\underline{\Theta}$ such that

$$\underline{\theta}_{ij} \leq \theta_{ij} \leq \overline{\theta}_{ij}. \tag{73}$$

To account for the stability criteria, the RCGA is formulated as a constrained optimization problem. This is achieved by imposing a constraint on the cost with a penalty function. Specifically, if the stability criterion is not satisfied, a multiplicative penalty is imposed on the cost such that

$$\text{if} \begin{cases} \max[\lambda_i(A_a)] < 1, & \mathcal{J} = \mathcal{J} \\ \text{otherwise,} & \mathcal{J} = penalty * \mathcal{J} \end{cases} \tag{74}$$

where $\lambda_i,\ i \in (1, 2, \ldots, m + 2n)$ are the eigenvalues of the augmented system A_a. Using this type of penalty helps to insure that, because of fitness values, individuals representing unstable systems will not survive the selection process. The penalty is chosen as 100 such that the unstable fitness values will be two orders of magnitude larger than their true values.

Illustrative Example of FDI for a Jet Engine

A numerical example is presented in this section to illustrate robust ℓ_1 estimator design using the Popov-Tsypkin multiplier and the application of the robust ℓ_1 estimator to robust fault detection of dynamic systems. The model used was supplied by NASA Glenn Research Center and is given as

$$x(k+1) = (A + \Delta A)x(k) + Bu(k) + D_{\infty,1}w_{\infty}(k) + R_a f_a(k), \tag{75}$$
$$y(k) = (C + \Delta C)x(k) + Du(k) + D_{\infty,2}w_{\infty}(k) + R_s f_s(k) \tag{76}$$

where the sampling period $T_s = 0.01$ sec. Only sensor faults are considered in this example, thus $R_a = 0$. The elements of the state vector $x \in \mathcal{R}^3$, are

$x_1 \triangleq$ High Pressure Spool Speed (rpm)

$x_2 \triangleq$ Low Pressure Spool Speed (rpm)

$x_3 \triangleq$ High Pressure Compressor Inlet Temperature ($^\circ C$).

The elements of the control input vector $u \in \mathcal{R}^3$, are

$$u_1 \triangleq \text{Main Burner Fuel Flow (kg/hr)}$$
$$u_2 \triangleq \text{Exhaust Nozzle Throat Area } (m^2)$$
$$u_3 \triangleq \text{Bypass Duct Area } (m^2).$$

The elements of the output vector $y \in \mathcal{R}^3$, are

$$y_1 \triangleq \text{Corrected High Pressure Spool Speed (rpm)}$$
$$y_2 \triangleq \text{Corrected Low Pressure Spool Speed (rpm)}$$
$$y_3 \triangleq \text{Corrected High Pressure Compressor Inlet Temperature } (^\circ C).$$

The variable w denotes a vector of disturbance signals.

The uncertainty matrices, ΔA and ΔC, are representative of some engine degradation over time. Thus, it is assumed that a newly constructed engine can be modeled with the nominal matrices A and C and with use, the parameters of the degraded engine are encompassed in the uncertainty. The system parameter matrices are

$$A = \begin{bmatrix} 0.9835 & 0.0110 & 0.0039 \\ 3.788e-4 & 0.9858 & 0.0026 \\ 4.230e-6 & -2.282e-4 & 0.9891 \end{bmatrix}, \quad D_{\infty,1} = \text{diag}\{0.1,\ 0.1,\ 0.01\}, \tag{77}$$

$$B = \begin{bmatrix} 0.0080 & 0.2397 & -0.0383 \\ 0.0068 & 0.1565 & 0.0248 \\ 2.691e-4 & -2.912e-4 & 2.558e-4 \end{bmatrix}, \quad R_s = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \tag{78}$$

$$C = \begin{bmatrix} 0.2383 & 0.4871 & 0.1390 \\ -1.074e-5 & -8.399e-4 & 3.784e-4 \\ 2.070e-5 & -4.132e-5 & -4.335e-6 \end{bmatrix}, \tag{79}$$

$$D = \begin{bmatrix} 0.4171 & -4.492 & 0.4875 \\ 7.968e-4 & -0.0050 & 2.861e-4 \\ -1.270e-5 & 4.837e-4 & -0.0021 \end{bmatrix}, \quad D_{\infty,2} = 0.1 \times I_{3\times 3}. \tag{80}$$

The uncertainty matrices $\Delta A = -H_A F_A G_A$ and $\Delta C = -H_C F_C G_C$, where

$$\begin{aligned}
& H_A = -\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad H_C = -\begin{bmatrix} 1 & 1 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \\
& G_A = \begin{bmatrix} I_{3\times 3} \end{bmatrix}, \quad G_C = \begin{bmatrix} I_{3\times 3} \end{bmatrix}, \\
& F_A = \mathrm{diag}\{\delta_{A_1},\, \delta_{A_2},\, \delta_{A_3}\}, \quad F_C = \mathrm{diag}\{\delta_{C_1},\, \delta_{C_2},\, \delta_{C_3}\},
\end{aligned} \tag{81}$$

with

$$\begin{aligned}
& -0.02167 \le \delta_{A_1} \le 0.02167, \quad -0.02174 \le \delta_{A_2} \le 0.02174, \\
& -0.02181 \le \delta_{A_3} \le 0.02181,
\end{aligned} \tag{82}$$

$$\begin{aligned}
& -0.01787 \le \delta_{C_1} \le 0.01787, \quad -0.03653 \le \delta_{C_2} \le 0.03653, \\
& -0.01043 \le \delta_{C_3} \le 0.01043.
\end{aligned} \tag{83}$$

Note that the uncertain parameters $\delta_{A_1} \ldots \delta_{A_3}$ correspond to parameter fluctuations in the diagonal elements of matrix A and $\delta_{C_1} \ldots \delta_{C_3}$ correspond to the first row of C. In the real-coded genetic algorithm, the chromosome string Θ (72) consisted of 39 genes, corresponding to the elements of $A_e \in \mathcal{R}^3$, $W \in \mathcal{R}^3$, $P \in \mathcal{R}^3$ and the diagonal elements of $H,\ N \in \mathcal{D}^6$. By using the objective function (57) with stability constraints (74), the respective gain and projection matrices are obtained for a bank of estimators. The nominal (uncertainty not considered) gain matrices were

$$W_{n,1} = \begin{bmatrix} -4.3677 & 0.9652 & 2.1343 \\ -46.331 & -0.4433 & 2.5012 \\ 185.10 & 0.0755 & -12.637 \end{bmatrix}, \quad P_{n,1} = \begin{bmatrix} 0.0324 & -0.1080 & 1.7946 \\ -0.0042 & 0.0218 & -0.0179 \\ 0.0242 & -0.0931 & 1.3789 \end{bmatrix}, \tag{84}$$

$$W_{n,2} = \begin{bmatrix} 8.5628 & 2.1809 & -0.9584 \\ -12.065 & 4.3910 & 1.2877 \\ 34.830 & 0.6572 & -3.4886 \end{bmatrix}, \quad P_{n,2} = \begin{bmatrix} -0.0072 & 0.3017 & 0.2845 \\ 0.0009 & -0.0376 & -0.1761 \\ 0.0092 & -0.3725 & -1.8386 \end{bmatrix}, \tag{85}$$

$$W_{n,3} = \begin{bmatrix} 11.665 & 2.2105 & -11.263 \\ -6.0256 & -1.1449 & -17.177 \\ 5.0019 & 0.9603 & -17.767 \end{bmatrix}, \quad P_{n,3} = \begin{bmatrix} -0.0041 & 0.3916 & -0.9689 \\ 0.0005 & -0.0207 & -0.2238 \\ -0.0025 & 0.3185 & -1.8625 \end{bmatrix}. \tag{86}$$

Note for all nominal filters the system matrix $A_e = A$. The robust (uncertainty explicitly considered) gain matrices obtained were

$$W_{r,1} = \begin{bmatrix} 0.0445 & 3.0049 & -0.0722 \\ -1.1029 & -0.7468 & 0.0282 \\ 12.8499 & 0.0867 & -0.1242 \end{bmatrix}, \; P_{r,1} = \begin{bmatrix} 0.0235 & 0.0430 & -0.2387 \\ -0.0072 & 0.2216 & 0.0108 \\ -0.0066 & -0.0337 & 0.0567 \end{bmatrix}, \tag{87}$$

$$A_{e,1} = \begin{bmatrix} 0.9049 & 0.0109 & 0.0040 \\ 0.0004 & 0.7968 & 0.0025 \\ 3.789e-6 & -0.0002 & 1.0687 \end{bmatrix}, \tag{88}$$

$$W_{r,2} = \begin{bmatrix} -0.0218 & 0.5019 & -0.0161 \\ 0.1665 & -250.65 & 0.0066 \\ 6.7877 & 0.0019 & -0.0030 \end{bmatrix}, \tag{89}$$

$$P_{r,2} = \begin{bmatrix} -4.143e-7 & 0.2373 & -2.0956 \\ -4.423e-7 & 0.0012 & 0.0062 \\ -2.894e-6 & -8.729e-5 & -0.0011 \end{bmatrix}, \tag{90}$$

$$A_{e,2} = \begin{bmatrix} 0.5480 & 0.0021 & -0.0003 \\ -1.057e-5 & 0.8245 & -0.0063 \\ 9.35980069e-6 & -0.0013 & 2.4357 \end{bmatrix}, \tag{91}$$

$$W_{r,3} = \begin{bmatrix} 9.1728 & 9.5942 & -1.8998 \\ -1.1983 & -3.0317 & -221.94 \\ -0.4734 & 0.7084 & -127.55 \end{bmatrix}, \; P_{r,3} = \begin{bmatrix} -0.0047 & 0.4759 & -0.3249 \\ -5.4336 & 0.0421 & -0.0067 \\ 0.0027 & -0.6124 & 1.36088 \end{bmatrix}, \tag{92}$$

$$A_{e,3} = \begin{bmatrix} 0.4731 & 0.0192 & 0.0075 \\ 0.0035 & 0.0955 & 5.180e-5 \\ -3.203e-6 & -1.691e-5 & 0.2037 \end{bmatrix}. \tag{93}$$

As shown above, for both the nominal and robust systems three filters were designed corresponding to targeted faults f_1, f_2 and f_3. To verify the solutions obtained by the RCGA, the frequency response of the closed-loop systems were examined. Specifically, Bode diagrams were used to check the magnitude of the transfer functions from the faults f_1, f_2, and f_3, and the disturbances w_1, w_2 and w_3 to the residual signals r_1, r_2 and r_3. Figure 4 shows the response of filter 3 of the nominal system, where the target fault is f_3 and the residual signal is r_3. It can be seen that the influence of the target fault signal on the residual is significantly larger than the influence of the nuisance faults and disturbances over all frequencies. Similarly, in Figure 5 filter 2 of the robust system, where the target fault is f_2 and the residual signal is r_2, the influence of the target fault signal on the residual is larger than the nuisance faults and disturbance signals.[2]

In order to illustrate the application of the robust ℓ_1 estimator to robust fault detection, FDI of the system in (75) and (76) subject to plant disturbances was performed. A bank of estimators (as described in Section 3) was designed for the set of y_i, $i \in \{1, 2, 3\}$, sensor outputs, i.e., the i^{th} estimator is designed to detect a fault in the y_i sensor while neglecting faults in the remaining sensors. Here the nominal case as well as the robust case are considered for the FDI process. Random white noise signals with zero mean were added as both the disturbance inputs and sensor noise. The variances of the disturbance inputs, w_1, w_2 and w_3, were 0.05, 0.08 and 0.03, respectively. In order to show the extent of robustness, uncertainty for all system matrices was considered. The uncertain parameters are assigned random values within their respective ranges. The values are given in Table 1.

This example only considered the occurrence of sensor faults within the system. A typical sensor fault in the jet engine is a drift in the sensor reading. Thus, a slow drifting (or ramping) sensor fault was added to a sensor reading at a particular instant in time. Specifically, the simulated fault signal can be described by the linear function

$$f_i(k) = \begin{cases} 0, & k < k_f \\ \tau(k - k_f), & k \geq k_f \end{cases} \tag{94}$$

where $\tau = 0.1$ is the slope and k_f is the instant at which the fault occurs. Due to the disturbance the finite-horizon infinity norm (64) of the residual with $N - N_0 = 60$ (corresponding to a time interval of 0.6 sec.), was nonzero even in the absence of faults.

In Figures 6 through 8 a single sensor fault was introduced in the system. Specifically, in Figure 6 a fault was introduced in sensor y_1 at $t = 20\,\text{sec}$, Figure 7 has a fault introduced in sensor y_2 at $t = 30\,\text{sec}$, while a fault in sensor y_3 is introduced at $t = 40\,\text{sec}$ in Figure 8. It can be seen that both the nominal and robust estimators were able to successfully detect and isolate each fault. This is evident as each faulty sensor the residual surpassed its respective threshold at the the time of occurrence of each fault. However, it is noted that in each nominal estimator system false alarms are given in one of the fault free sensors. This is due to the fact that uncertainty was not accounted for in the design of these estimators. These false alarms are avoided with the robust filters. In Figure 9 multiple faults were introduced in sensors y_1, y_2 and y_3 at $t = 25$ sec, $t = 10$ sec and $t = 40$ sec, respectively. It can be observed that in this instant both nominal and robust systems were able to isolate each target fault from the other nuisance faults. A false alarm is again given as the first residual surpasses its threshold well before a fault is introduced in the sensor. This does not occur

[2] These trends are representative of the behavior of each filter response.

with the robust estimator.

Conclusions

This paper considered the application of robust ℓ_1 estimation for uncertain, linear discrete-time systems to the robust fault detection and isolation. Mixed structured singular value theory of[18] was used to design a bank of robust ℓ_1 estimators and the resulting fixed threshold logic. By considering a discrete, linear model of a jet engine with real parametric uncertainties and introducing drifting sensor faults, it was shown that the robust FDI methodology based on fixed thresholds was capable of detecting and isolating failures in each of the particular sensors. Also, by designing the robust estimators to explicitly account for uncertainty false alarm rates were significantly reduced.

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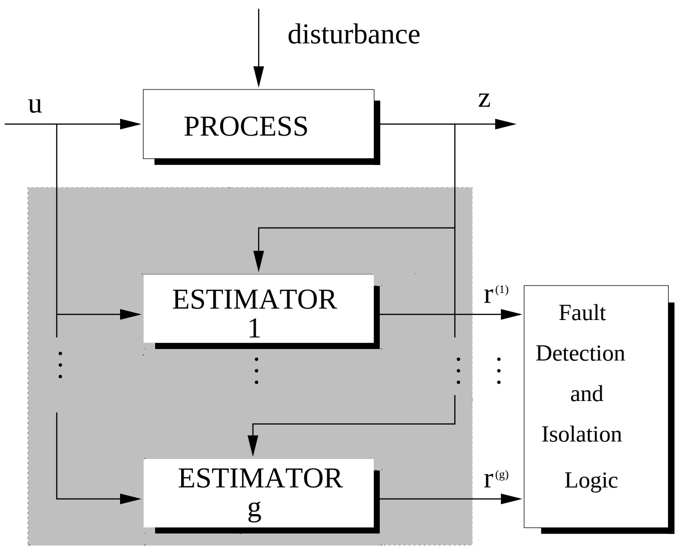
List of Table Captions

	Figure 6	Figure 7	Figure 8	Figure 9
δ_{A_1}	0.004217	-0.000386	0.003415	-0.010360
δ_{A_2}	-0.011812	-0.007753	-0.009804	-0.007485
δ_{A_3}	0.007793	0.003931	0.006159	-0.000564
δ_{C_1}	0	-0.010086	-0.012755	-0.009738
δ_{C_2}	0	0.011859	0.018581	-0.001702
δ_{C_3}	0	-0.000169	0.001494	-0.004532

List of Figure Captions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Isolation
Logic

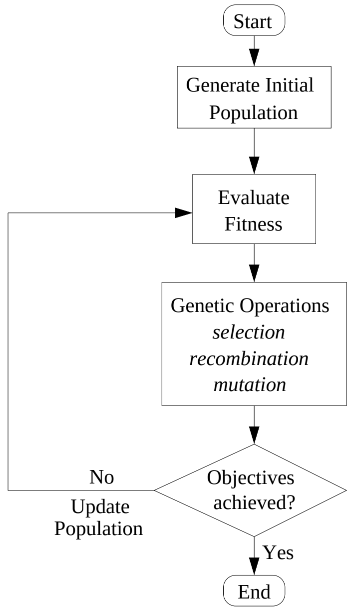
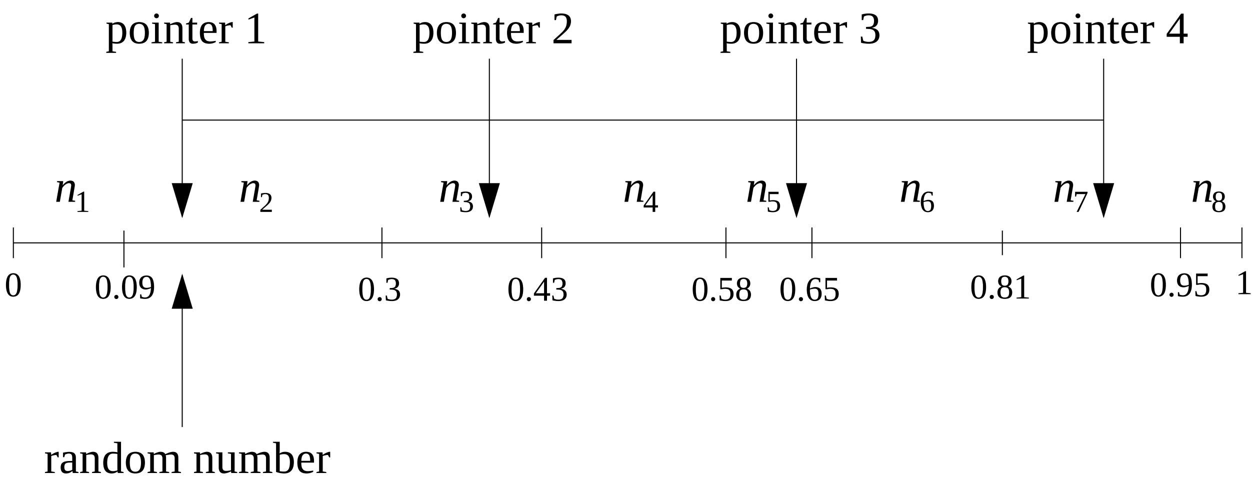
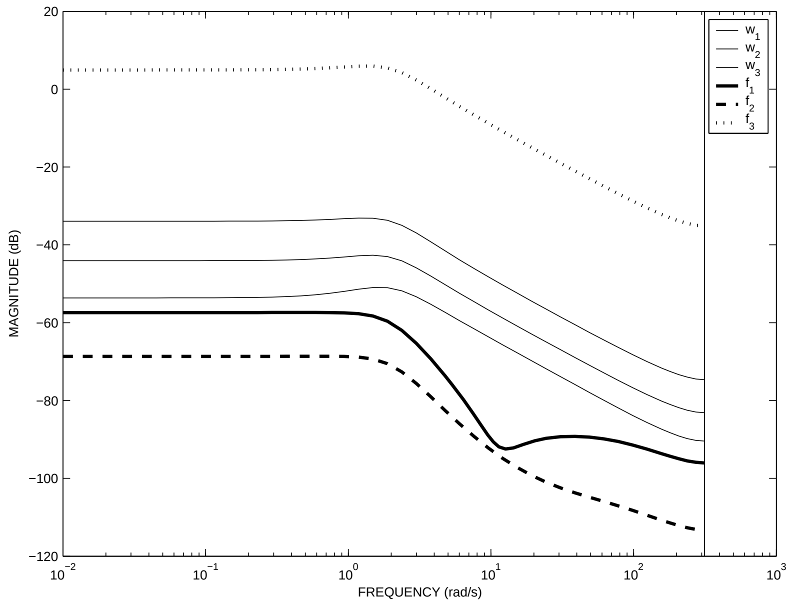

Start
Generate Initial Population
Evaluate Fitness
Genetic Operations
selection
recombination
mutation
Objectives achieved?
No
Update Population
Yes
End

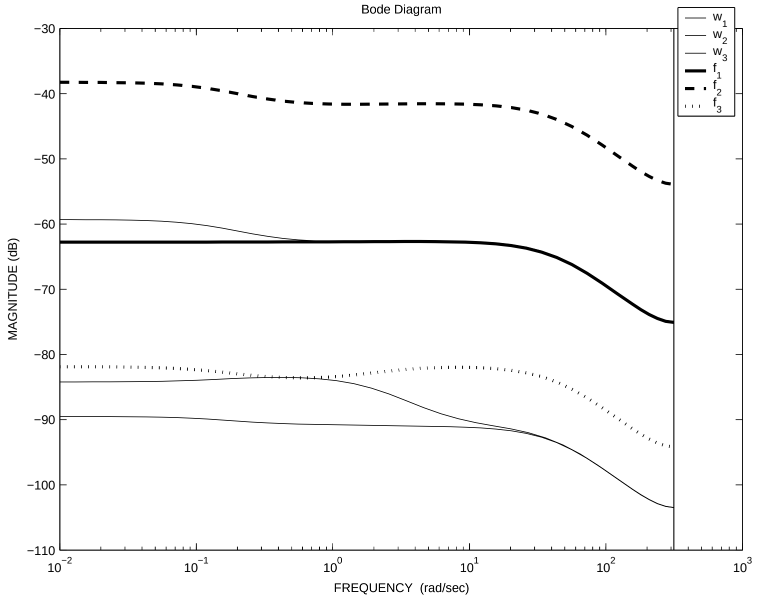
Figure 2:

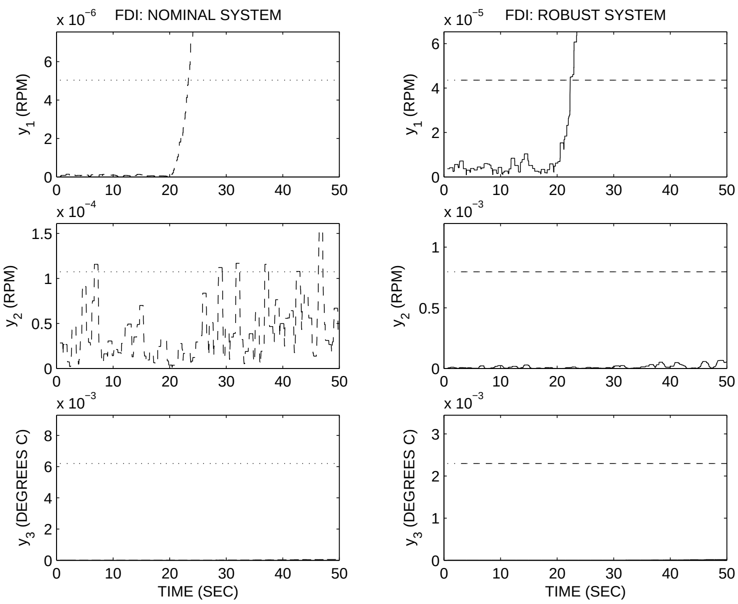

pointer 1
pointer 2
pointer 3
pointer 4
n_1
n_2
n_3
n_4
n_5
n_6
n_7
n_8
0
0.09
0.3
0.43
0.58
0.65
0.81
0.95
1
random number

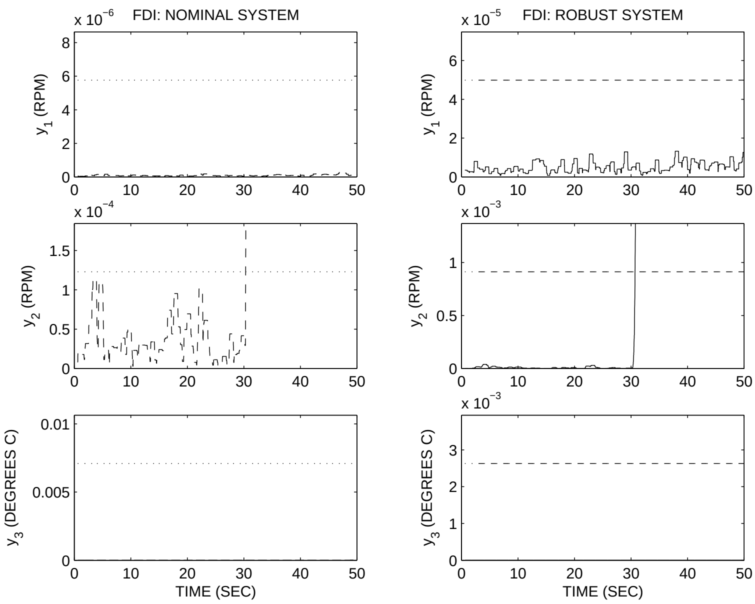


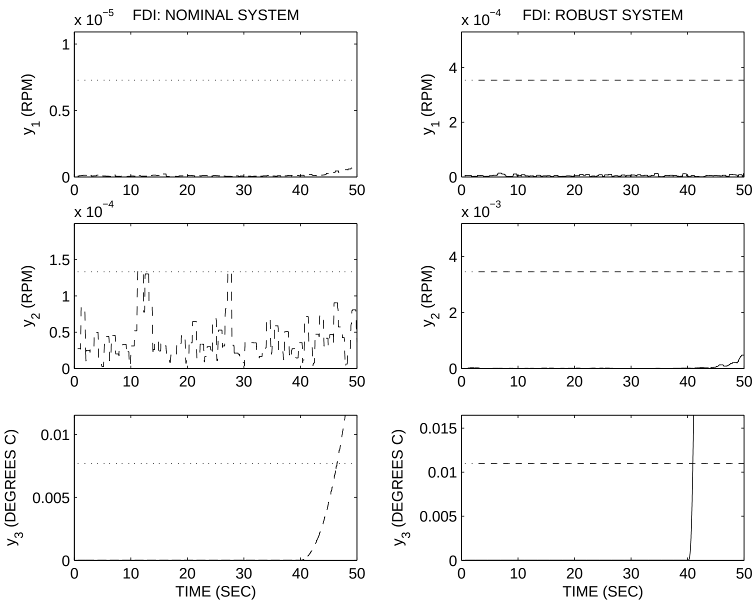
20
0
−20
−40
−60
−80
−100
−120
MAGNITUDE (dB)
10^{-2}
10^{-1}
10^{0}
10^{1}
10^{2}
10^{3}
FREQUENCY (rad/s)
w_1
w_2
w_3
f_1
f_2
f_3

 Figure 4:

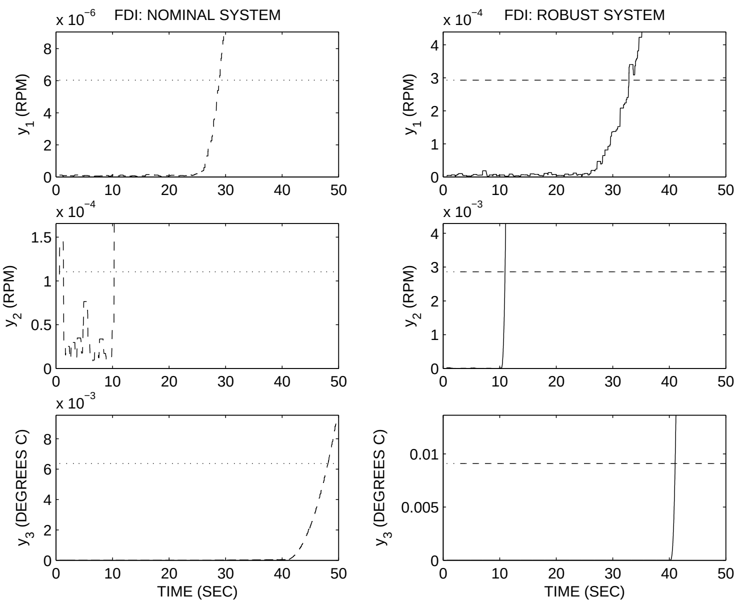

Bode Diagram
w1
w2
w3
f1
f2
f3
−30
−40
−50
−60
−70
−80
−90
−100
−110
MAGNITUDE (dB)
10−2
10−1
100
101
102
103
FREQUENCY (rad/sec)


FDI: NOMINAL SYSTEM
FDI: ROBUST SYSTEM
y_1 (RPM)
y_2 (RPM)
y_3 (DEGREES C)
x 10^{-6}
x 10^{-5}
x 10^{-4}
x 10^{-3}
TIME (SEC)


FDI: NOMINAL SYSTEM
FDI: ROBUST SYSTEM
x 10^−6
x 10^−5
x 10^−4
x 10^−3
x 10^−3
y1 (RPM)
y2 (RPM)
y3 (DEGREES C)
TIME (SEC)
TIME (SEC)



FDI: NOMINAL SYSTEM
FDI: ROBUST SYSTEM
x 10^-5
x 10^-4
x 10^-3
y1 (RPM)
y2 (RPM)
y3 (DEGREES C)
TIME (SEC)


FDI: NOMINAL SYSTEM
FDI: ROBUST SYSTEM
x 10^−6
x 10^−4
x 10^−3
y1 (RPM)
y2 (RPM)
y3 (DEGREES C)
TIME (SEC)