Solutions should be fully derived showing all intermediate results. Explain all reasoning. One book of mathematical tables, such as Schaum’s Mathematical Handbook, may be used, as well as a calculator, and a handwritten single-sheet letter-size formula sheet. Various appendices of the book are attached.

1. Consider a viscous fluid outside a solid circular pipe of radius $a$.
   (a) Without doing any mathematics, just using your knowledge of ideal flows, explain why if the pipe is rotating, there is a viscous flow field that satisfies the no-slip conditions at the surface of the pipe and whose velocity is zero at infinite distance from the pipe.
   (b) But now show, using mathematics, that if the pipe is translating along its axis with a speed $V_0$, there is no flow field that satisfies the no slip condition and that is zero at infinite distance. At least not one that satisfies the conditions
   \[ v_r = v_z(r, \theta, z, t) \quad v_\theta = 0 \quad v_z = v_z(r) \quad p = p(r, \theta, t) \]
   Do not assume anything more than the above, all else must be derived. Use an effective pressure to deal with any possible direction of gravity and give the final most general possible (true) pressure field.

2. Discuss the ideal flow $F = tz^{3/2}$ for times $t > 0$.
   (a) What is the shape of the streamlines? Draw a comprehensive set of them neatly.
   (b) What sort of physical flows might this correspond to? (There are two plausible possibilities.)
   (c) What is the magnitude and direction of the velocity on the positive $x$-axis? Comment on the boundary conditions if the $x$-axis corresponds to a solid wall. Are the ones you expect to be satisfied really satisfied?
   (d) Is there a stagnation point in the flow? If so, where is it?
   (e) Find the pressure at any arbitrary location in the field. Take the stagnation pressure to be a given $p_0$. Ignore gravity.

3. Discuss the unsteady boundary layer flow around an ellipse that is impulsively set into motion. You cannot do the mathematics without a computer, but you should be able to formulate the mathematical problem. In particular, address the following items:
   (a) Explain what are the appropriate boundary layer variables to use in the formulation of the mathematical problem, and exactly how they are defined. Include a very neat sketch of them.
   (b) List the governing equations for the dependent variables.
   (c) List the boundary conditions to be satisfied.
   (d) Solve one equation.
   (e) For inhomogeneous boundary conditions, how would you get the values of the boundary condition? You do not have to find the actual values, but indicate precisely how you can get them, exactly what you will have to do. There should be no ambiguity in what the quantities to find are.