Dynamic Legged Robots For Use In Multiple Regimes: Scaling, Characterization And Design For Multi-Modal Robotic Platforms

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DYNAMIC LEGGED ROBOTS FOR USE IN MULTIPLE REGIMES:
SCALING, CHARACTERIZATION AND DESIGN FOR MULTI-MODAL ROBOTIC PLATFORMS

By

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A Dissertation submitted to the Department of Mechanical Engineering in partial fulfillment of the requirements for the degree of Doctor of Philosophy

Degree Awarded:
Summer Semester, 2013
Bruce D. Miller defended this dissertation on June 28th, 2013.

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ACKNOWLEDGMENTS

There are several people whom I would like to thank for their assistance and support in helping me through this process. First, I would like to thank my advisor, Dr. Jonathan Clark, who has been an integral part of all of my studies and achievements since beginning my graduate work. His guidance and mentorship both in the classroom and laboratory have helped me progress as a student, teacher, and researcher and truly made it possible for me to reach this point in my development. He has challenged me to continually strive to make the most of my opportunities and I am greatly appreciative of all the support and advice he has provided.

I also would like to thank Dr. Emmanuel Collins, Dr. William Oates and Dr. Rodney Roberts, who have served as members of my dissertation committee. The conversations I have had with each of them have helped shape my graduate work and provided several insightful vantage points from which to consider and apply the findings of this work. Their feedback has been invaluable and greatly contributed to the success of my studies.

In addition, I am grateful for the assistance that has been provided by both present and former colleagues in the lab. Ben Andrews developed the single-legged hopping platform used in my stability studies and helped in acclimating me to the lab when I first arrived. Camilo Ordonez developed the electronics package used on SCARAB and lent support for both electrical debugging and the development of a software package. Chris Kulinka and James Dickson both provided advice and constructive criticism that proved crucial in getting SCARAB operational. Jason Newton has been integral as well, providing support and assistance in the design and carrying out experiments to test the platform capabilities. Several other colleagues have played roles in the development and experimentation of SCARAB, including Jae Yun Jun, Michael Bunne, Asa Darnell and Bryan Rodgers.
I would finally like to thank my family and friends who have always been supportive of my studies. In particular, I would like to acknowledge my parents, Lori and Wayne Miller, my brothers, Gary and Alex Miller, my grandmothers, Harriet Miller and Flora Fine, and my fiance, Emily Wehr, for the love, encouragement, and support they have always provided me.
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ABSTRACT

Animals have demonstrated that legged locomotion can provide an efficient, rapid and robust means for traversing natural and artificial terrains and obstacles while employing several distinct locomotion modalities. This has led researchers to develop biologically-inspired, legged systems that improve the mobility of robotic platforms on rough terrain as compared to traditional wheeled and tracked systems. While several effective, dynamical legged robots have been developed, they still are a step behind their biological counterparts in terms of speed, efficiency, and, in particular, versatility. A contributing factor is that these platforms are designed and optimized to utilize a single locomotion modality and are not capable of traversing the variety of terrains found in the natural world, whereas many biological creatures are capable of multi-modal locomotion. This motivates the exploration of the principles that will enable the development of biologically-inspired, dynamical legged robots capable of utilizing multiple locomotion modalities.

This dissertation focuses on several topics related to the development of biologically-inspired, multi-modal platforms. In particular, a focus is made towards the extension of a dynamic scaling method for dynamical legged systems, the assessment of dynamic stability metrics for use on experimental platforms and the design and characterization of the first legged, robotic platform capable of dynamical, biologically-inspired locomotion in multiple domains. The resulting insights from this work will foster an improved understanding of multi-modal robotic locomotion and provide a set of tools for designing and characterizing future robotic platforms.
CHAPTER 1

INTRODUCTION

1.1 Motivation

As the use of robots in commercial, industrial, and military applications expands, several emergent challenges have been encountered. A key hurdle among these is mobility, particularly with regards to maintaining speed and stability while minimizing power consumption when faced with varied and changing environments. Traditional mobility mechanisms for robots, such as wheels and tracks, are well equipped for moving efficiently and at high speeds on prepared, artificial surfaces, but may quickly become mired when faced with mud, rocks, or other obstacles [1, 2]. While common engineered solutions may fail in such circumstances, a cursory observation of biological locomotion strategies demonstrates that legs provide a reliable means of traversing these environments. Legged animals can be found moving across almost any surface on the planet. As such, they present an ideal archetype for robotic locomotory systems.

When considering the mobility afforded by legs, several distinct advantages can be identified, as depicted in Fig. 1.1. The first is that they only require isolated footholds as opposed to the need for a continuous path of supports [3]. This allows legs to function at near-peak performance on terrains with a limited number of ‘good’ footholds while wheeled or tracked platforms require a continuous path and may suffer from degraded performance if faced with highly-broken terrain. A second advantage of legs is that due to the use of discrete contacts, legged platforms can simply step over obstacles that traditional mobility systems would need to climb over or go around [3]. This allows them to operate in a similar fashion on both rough and level terrains, allowing for similar speed and efficiency in rough environments.
as in a prepared setting. A final advantage is the presence of zero-delay, mechanical feedback systems, or *preflexes*, that allow for a rapid stabilizing response to system disturbances [4]. This is instantiated in biological systems via the intrinsic force-length and force-velocity properties of biological tissues, such as muscles and tendons. Together, these properties of legged locomotion provide several reasons to be considered over traditional mechanisms.

While the potential advantages of legs as a mean of locomotion provides an incentive for their widespread utilization, few highly successful legged platforms have been developed. One reason for this is that no straight-forward approach for instantiating ‘bio-like’ locomotion on a robotic platforms exists. One technique that has been explored is the explicit use of biomimicry, which aims to reproduce biological phenomenon through imitation of natural structures and processes [5]. However, the complexity of biological systems makes such attempts to rigorously reproduce the biological processes a daunting task. While researchers have investigated the use of biomimetics, resulting platforms are cumbersome and difficult to control due to the large number of actuated degrees-of-freedom, making coordinated motion a significant challenge [5, 6]. An alternative approach is to consider abstract biological models and examine the whole-body dynamics rather than detailed, physiologically accurate representations of specific legged animals. This has led to the formulation of reduced-order models that encapsulate the simplified body dynamics of these creatures [7].
Various legged locomotion modalities have been modeled in this way, including walking [8], running [9,10], and climbing [11].

Using these models as templates for dynamical legged locomotion, researchers have developed several robots that are capable of emulating animal-like behaviors, particularly at high speeds [12–16]. But while these platforms have shown a degree of success, their locomotion capabilities still severely lag their animal counterparts. One reason for this disparity is that while these platforms are capable of overcoming certain challenging situations, they are largely restricted in the locomotion modalities available to them. When observing animals in nature, one may see a creature run across the ground, leap to a nearby tree, and quickly climb to a safe perch. This proficiency in utilizing multiple locomotion modalities is beyond the capabilities of currently developed platforms. However, as our understanding of the underlying dynamics of these different locomotion modalities increases, we will soon have the tools available to develop effective, multi-modal systems.

1.2 Problem Statement and Approach

As previously stated, with the increased utilization of robots in various applications, a corresponding improvement in their capacity to traverse difficult and diverse environments is needed as well. Furthermore, the proficiency shown by animals to adroitly negotiate a wide array of challenging and changing terrains supports the use of their locomotion strategies as a paradigm for robotic mobility. Thus, the primary objective of this research is to foster an understanding of the principles that will enable the development of biologically-inspired, dynamical, legged robots capable of stably traversing rough terrains and utilizing multiple locomotion modalities.

To achieve this goal, several aspects that contribute to this understanding will be considered. First, due to the potential discrepancy between the size of the representative animal models and the desired size of the final platform, it is necessary to have a means by which the system parameters can be scaled to the desired size while preserving the biologically-inspired locomotion characteristics. Though rudimentary procedures for such scaling exist, they provide overly restrictive scaling constraints that limit the design flexibility. Thus, the first thrust of this dissertation is to formulate a scaling procedure derived \textit{ab initio} and
demonstrate its efficacy in preserving the dynamical characteristics of legged locomotion models.

A second concern with biologically-inspired, multi-modal platforms, as well as dynamical legged platforms in general, is that while speed and efficiency can be readily measured, characterizing such a system’s stability can prove problematic. Not only does stability have many potential implications within the context of dynamical platforms, but systematic ways of measuring stability experimentally are lacking. This motivates the second thrust of this dissertation, which is to determine an effective and practical means of quantifying the stability of experimental dynamical platforms.

The final component of this dissertation is the development of a biologically-inspired, multi-modal platform capable of running on level and vertical surfaces. While previous dynamical platforms have been developed in each individual domain, this effort represents the first instantiation of a physical, legged robot that is capable of high-speed locomotion in multiple domains. This platform will be used to consider the adaptations that are required to transition between these two regimes, as well as to characterize the behavior and consider the performance in the respective domains relative to the inspirational models.

1.3 Contributions

The primary aim of this dissertation is to consider the unique mobility advantages afforded by multi-modal, biologically-inspired, legged systems and consider a set of tools for the development of such a platform. As such, several contributions have been established via this dissertation including the following:

1. Derivation of a general set of dynamic scaling laws and validation that they preserve the dynamic locomotion characteristics of legged systems, both in simulation and experimentally, as well as consideration of the potential implications afforded by use of the dynamic scaling relations.

2. Determination of an appropriate metric for quantifying disturbance rejection of dynamical, legged systems that is well-suited for both experimental and simulation applications.

3. Development of a biologically-inspired, legged robotic platform that demonstrates multiple dynamical locomotion modalities.
4. Characterization of the locomotion performance of the multi-modal platform and exploration of the trade-offs and modifications necessary to utilize both modalities interchangeably.

5. Illustration of the similarity between the locomotion dynamics of the multi-modal platform and the inspiring locomotion templates.

1.4 Dissertation Outline

The remainder of this dissertation is organized into six chapters. Chapter 2 reviews previous work on many of the topics related to the research performed as part of this dissertation. Section 2.1 addresses the evolution of biologically-inspired templates, in particular focusing on models of high-speed, legged locomotion. Section 2.2 follows with a brief review of previously developed legged robots, with an emphasis on those that aim to anchor the aforementioned biologically-inspired templates. In Section 2.3, several of the design challenges related to the development of these and other legged platforms are considered. Section 2.4 concludes the review various means for characterizing robot performance.

Chapter 3 considers scaling considerations for dynamical systems. First, a set of dynamic scaling relations are derived and compared to previous efforts. This is followed by a series of simulation studies of several dynamical legged systems to confirm the applicability of the derived scaling laws. Additionally, the implications of dynamic scaling on the design of robotic platforms is considered, with particular attention focused on actuator and materials selection as well as the preservation of dynamic similarity when transitioning between environments with different gravitational effects.

Chapter 4 focuses on the quantification of dynamic stability, particularly in an experimental setting. The chapter begins with a review of several stability metrics that have previously been considered for dynamical systems in some context. Next, simulation and experimental inquiries are presented to assess the efficacy of the various metrics and compare them against each other. These studies are used to identify a subset of metrics that can be used in a predictive manner on both in both experimental and simulation settings to quantify dynamic stability of dynamical legged systems.

In Chapter 5, the design of a dynamical, multi-modal platform, SCARAB, is presented. This begins with a description of the system and preliminary simulations involved in its de-
velopment. A particular emphasis is placed on highlighting the parameter changes necessary for transitioning between running and climbing modalities. Next, studies of the platform’s running and climbing behavior are presented and the performance of the platform is compared to the underlying templates.

Chapter 6 concludes this dissertation by summarizing the findings of this work and suggesting avenues for future investigation.
2.1 Biologically Inspired Locomotion Models

As the use of robots in challenging and diverse settings increases, it is natural to look to animals as effective mobility archetypes and attempt to distill the factors that contribute to their successful locomotion. One such approach is to consider models that capture the full-body dynamics of legged locomotors in simple, low-dimensional formulations [7]. Analysis of these models can reveal general locomotion principles that are often common to animals of varying sizes and morphologies [17]. Additionally, the abstract representation of these models provides flexibility in the design of robotic platforms that aim to emulate the biologically-inspired behaviors, as internal forces and motions are not captured nor are they relevant in producing similar performance. This approach has been used to characterize several modalities of biological locomotion, including walking [8,18,19], running [9,10,20], and climbing [11]. As the aim of this dissertation is related to the development of a high-speed, legged platform, the discussion will be restricted to high-speed, legged models (i.e. running and dynamic climbing), though the general principles developed and discussed herein could be extended to address other locomotion modalities as well.

2.1.1 Spring-Loaded Inverted Pendulum Model

The spring-loaded inverted pendulum (SLIP) model has been widely used to describe the sagittal plane dynamics of running. The model was conceived from the observation that even with drastic variations in leg number, size, and morphology, animals demonstrate similar energetics, gaits, stride frequencies, and ground reaction force patterns while running [17]. In its simplest formulation, the SLIP model consists of a point-mass $M$ that is affixed to a
massless leg. The leg is modeled as an axially elastic, transversely rigid, linear spring with a stiffness of \( k \) and a nominal length of \( l_0 \). At the distal end of the leg is a foot, which contacts the ground and acts as a moment-free pin joint during stance to anchor the leg to the running surface.

The locomotion dynamics captured by the SLIP model are restricted to the sagittal plane, as depicted in the sample stride shown in Fig. 2.1. Each stride starts when the foot establishes contact with the ground at a touch-down (TD) event, beginning the stance phase. At touch-down, the leg spring is extended to its nominal rest length and is oriented at the touch-down angle \( \beta_{TD} \), measured clockwise from the horizontal inertial axis to the leg axis. The velocity and heading of the point-mass are expressed as \( v \) and \( \delta \), respectively. During the first half of stance, the leg compresses under the momentum of the point-mass and gravity. During the second half of stance, the leg extends, returning the stored elastic potential, which results in a lift-off event when the ground reaction force drops to zero,
breaking the contact between the foot and the ground and initiating the flight phase. The flight phase persists, with the body trajectory being governed by simple ballistic dynamics, until the foot once again contacts the ground at the next touch-down event.

The governing equations for stance and flight phases can be simply derived as follows. As shown in Fig. 2.1, when the model is in stance, a polar coordinate frame \((\zeta, \theta)\) with the origin at the foot-pivot is used, while the Cartesian inertial frame \((x, z)\) is used to describe the trajectory during flight. For stance, the equations of motion are derived from the Lagrangian

\[
L = \frac{M}{2} \left( \dot{\zeta}^2 + \zeta^2 \dot{\theta}^2 \right) - Mg \zeta \cos \theta - \frac{k}{2} (\zeta - l_0)^2,
\]

(2.1)

which, by using the Euler-Lagrange equations, yields

\[
\ddot{\zeta} = \zeta \dot{\theta}^2 - g \cos \theta - \frac{k}{M} (\zeta - l_0)
\]

\[
\ddot{\theta} = -2 \dot{\theta} \dot{\zeta} + g \sin \theta.
\]

(2.2)

The flight dynamics, governed by simple ballistic motion, can be simply written as

\[
\ddot{x} = 0
\]

\[
\ddot{z} = g,
\]

(2.3)

where \(g\) is the gravitational acceleration acting on the point-mass.

### 2.1.2 Lateral Leg Spring Model

While the sagittal plane dynamics of biological runners have been widely observed to resemble those of SLIP, this model only captures the dynamic behavior in the fore-aft and vertical directions, neglecting any lateral motions in the system. While this assumption may be within reason for large animals with an upright posture, small runners with splayed legs, such as insects and amphibians, tend to generate significant lateral forces and motions when running [21]. To capture the horizontal plane dynamics of these biological runners, the Lateral Leg Spring model was developed [10]. The model consists of a body with a mass of \(M\) and a moment of inertia of \(I\) that is attached to two axially elastic, transversely rigid legs with a stiffness of \(k\) and a nominal rest length of \(l_0\). The legs are attached to the body at a prescribed distance \(d_1\) from center of mass along the longitudinal body axis.
and $d_2$ to either side of the longitudinal body axis via a freely rotating pin joint. As in the SLIP model, the distal ends of the legs act as feet, creating a moment-free pin joint when attached to the running surface.

Similarly to SLIP, the dynamic locomotion utilizing LLS is formulated as a hybrid dynamic system. However, instead of being split between stance and flight phases, the LLS model is always in stance, with the hybrid nature arising from the left and right legs alternating contact with the running surface. A stride begins when one leg establishes contact at a touch-down event as the foot engages the ground. At this point, the center of mass has a velocity magnitude and heading angle of $v$ and $\delta$, respectively. The leg spring then compresses under the momentum of the system, storing the kinetic energy as elastic potential that is then returned as the spring re-extends. Note that gravitational acceleration is not a factor since it is acting perpendicular to the horizontal plane. When the ground-reaction force at the foot drops to zero, the stride phase ends, with the stance leg switching to the opposite leg, which has been set to touch-down with an angle of $\beta_{TD}$. The same behavior is exhibited during this phase as in the previous. This pattern is repeated to produce the horizontal plane running. A sample of the trajectory generated through running with the LLS model is shown in Fig. 2.2. Although energetically conservative, the hybrid-dynamics of LLS, like SLIP, exhibit self-stabilizing behaviors when properly tuned.

### 2.1.3 Full-Goldman Climbing Model

As with level ground running, animals of varying size, leg number, attachment mechanism, and morphology have demonstrated the capacity for rapid vertical locomotion. While it would seem that differing climbing dynamics would be adopted for these animals, the center of mass trajectories and ground reaction force profiles show similar characteristics [11]. Two key points can be extracted from the study of these rapid climbers. First, the center of mass trajectories appear to be pendular even though climbing animals typically have multiple legs in contact with the climbing substrate at any given time, restricting free pendular dynamics. Second, significant lateral forces are generated, almost half the magnitude of those generated in the fore-aft direction, indicating that lateral dynamics play a crucial role in rapid vertical locomotion [16].
Figure 2.2: General trajectory of the LLS model for a single stride. The system has a body of mass $M$ and moment of inertia $I$ attached to a spring of stiffness $k$ and rest length $l_0$, which is offset $d_1$ vertically and $d_2$ horizontally from the center of mass of the body. When touch-down occurs, the new stance leg is extended to its rest length and has a sprawl angle of $\beta^{TD}$. The center of mass velocity magnitude and heading angle are given by $v$ and $\delta$, respectively. During each leg’s stance phase, the governing equations are written in terms of the polar coordinates $l$ and $\theta$, with the origin placed at the foot pivot.
Figure 2.3: Schematic and general trajectory of the FG model. (A) shows a schematic of the model. The parameters $M$, $I$, $d_1$, $d_2$, $k$, $l_0$, and $\beta$ represent the system mass, the system inertia, the vertical shoulder offset, the horizontal shoulder offset, the leg stiffness, the leg rest length, and the leg sprawl angle, respectively. The linear actuation element is used to inject energy into the leg spring. (B) shows a sample trajectory of the model of a single stride, consisting of a left and right step.
The dynamics generating this climbing behavior are captured in the reduced-order Full-Goldman (FG) model for dynamic climbing, a schematic of which is shown in Fig. 2.3A. The model is composed of a rigid, distributed mass body of mass $M$ and moment of inertia $I$ attached to two massless legs with a nominal length of $l_0$ consisting of a spring of stiffness $k$ in series with a linear actuation element. At the distal end of each leg is a foot that can attach to the climbing substrate as a moment-free pin joint, as with the SLIP and LLS models. The proximal ends of the legs are fixed a distance $d_1$ above the center of mass and $d_2$ to the left and right. The hips, connecting the legs to the body, are rotationally rigid attachments, locking the legs at a prescribed sprawl angle $\beta$, defined as the angle between the longitudinal body axis and the axis intersecting both the hip and the foot of each leg.

During climbing, each step begins with a touch-down event, which occurs when one foot establishes contact with the climbing surface, beginning the stance phase for that leg. At this point the leg is at its rest length and the linear actuation element is maximally extended. The actuation element then begins to contract, pulling the body towards the stance foot pivot as the body swings like a pendulum about this point. While the stance leg contracts, the actuation element of the opposing leg extends to the maximal length to prepare for the next step. The stance foot maintains contact with the climbing substrate until the stance leg actuation element has fully contracted, at which point the stance foot breaks contact with the climbing substrate while the opposing foot establishes attachment and begins the next step. This leg follows the same behavior until the actuator is again fully contracted, triggering the next step transition. The period between two touch-down events of the same leg is defined as a stride. The model continues to take alternating steps to generate upward vertical motion.

### 2.2 Anchoring Biological Models in Robotic Platforms

Many studies have been conducted on the models described in Section 2.1 in an effort to understand the effects of various system parameters on locomotion performance as well as to devise control approaches that will improve performance for both these simple models and more complex, derivative models. However, the eventual goal of these inquiries is to field robots capable of instantiating the biological templates [7] and serving as effective
mobility systems. Though there is currently no formulaic method to anchor these reduced-order models to a robotic instantiation, several robots have been developed that emulate the model dynamics with varying degrees of success. In the following sections, several of these platforms are considered and the successes and challenges which they encountered are reviewed.

2.2.1 Running Robotic Platforms

While legged robots have been in existence since the early 1900’s, the first platforms to exhibit dynamical, animal-like behaviors were not produced until the late 1970’s, when Matsuoka developed a mechanism that could balance while hopping in a plane [22]. Raibert built off this work to develop several hopping and running platforms, which included one, two, and four legged systems that were able to run both on a boom and unsupported and demonstrated limited success at navigating known cluttered environments [3,23,24]. While these platforms demonstrated qualitatively animal-like behaviors, reduced-order running models, such as SLIP, had not yet been explicitly defined and no formal comparison of the similarity to animal locomotion was made.

In the following years, a number of platforms have been developed which explicitly consider and exploit biological locomotion principles, which are well summarized by Sayyad, et al. [25] and Zhou and Bi [26]. Some of the more prominent platforms to be designed with the biologically-inspired templates in mind include RHex [12], iSprawl [27], and Scout II [15]. These platforms have collectively demonstrated the effectiveness of using simple, underactuated designs to produce robust, high-speed locomotion while exploiting the underlying dynamics of the biologically-inspired templates. All of them are able to operate untethered for extended periods with no more than a single actuator per leg, and in the case of iSprawl, only one drive motor for the entire platform. And while iSprawl and Scout II are limited in their ability to traverse rough terrains and obstacles, RHex is quite adept at navigating many terrestrial challenges, including rock piles, stairs, sand, and mud [28,29]. These platforms have demonstrated that legged robotic systems are rapidly progressing towards providing the speed and reliability of their biological counterparts on rough terrain.
2.2.2 Climbing Robotic Platforms

Legged climbing platforms have only been examined for a brief period compared to terrestrial legged robots, and this is particularly true for dynamical climbing platforms. Of these platforms, which include ROCR \[30\], ParkourBot \[31\], and DynoClimber \[16\], only DynoClimber (and its derivatives, ICAROS \[32\] and BOB \[33\]) attempt to anchor a biologically-inspired climbing template. Even so, DynoClimber has done so with great success, climbing at up to \(0.66 \, \text{m} \, \text{s}^{-1}\) and showing an impressive resilience to missed foothold. While there are few examples of biologically-inspired, dynamical robotic climbers, these few have shown significantly improved locomotion over previously developed climbing platforms.

2.2.3 Multi-Modal Robotic Platforms

As the usage of robots increases, researchers have considered incorporating multiple modalities on a single platform to improve the potential applications. While the platform developed as a part of this dissertation is the first system to demonstrate multiple dynamic legged modalities in different domains (on level and vertical surfaces, in this case), several other legged multi-modal robots have been previously developed\(^1\). The first type, which have received the greatest attention, are those that are able to both walk and run (and oftentimes jumping as well). In theory, the majority of multi-legged running platforms, including those presented in Section 2.2.1, are able to exhibit walking behaviors as well, though the walking modality is not as well quantified. However, there are several studies of platforms that have explicitly demonstrated both walking and running or walking and jumping behaviors \[37–40\]. Furthermore, a few platforms have recently shown a limited capacity to exhibit both running and jumping behaviors \[41,42\].

A second type of commonly considered multi-modal systems are wheel-legged hybrids. This includes platforms with wheels on the ends of articulated limbs \[43–45\] as well as those which can transform there legs into wheels and vice versa \[46–49\]. These platforms aim to leverage the improved efficiency of wheeled systems on smooth terrains while maintaining the ability to negotiate more challenging environments afforded by legs.

\(^1\)A number of non-legged, multi-modal robots have been developed as well, including numerous snake robots \[34–36\].
Two other types of legged multi-modal systems have been previously considered. The first, which include ICAROS [32] and MLAV [50], attempt to combine legged and winged locomotion on a single platform. A second type are amphibious platforms, demonstrated by the AQUA [51] and SeaDog [52] robots.

2.3 Scaling Considerations for Biologically-Inspired Models and Robots

While a number of biologically-inspired robots have been fielded, many challenges deter roboticists from widely adopting these platforms. One challenge is scaling systems such that they inherit the locomotion characteristics of their often much smaller archetypes. For theoretical analysis of legged systems, non-dimensionalized models are commonly utilized to avoid issues of scale [53, 54]. While this serves an effective simulation tool and reduces the number of independent system parameters, the process can be cumbersome and the non-dimensionalized results may not make intuitive sense without re-dimensionalization. Furthermore, a method for developing a scaled version of a previously existing platform via non-dimensionalization is poorly defined.

An alternative approach is to utilize dynamic scaling. This technique uses a set of scaling relations that preserve the geometric, kinematic, and dynamic similarity between an original, unscaled model and a final, scaled version of a given system [55]. Dynamic scaling relations remove the need to non-dimensionalize and re-dimensionalize at scale and, additionally, do not even require a dynamic model, a particularly beneficial trait when developing a scaled version of a physical system. The use of dynamic scaling has several benefits that contribute to its fit in scaling situations, including the preservation of all non-dimensional locomotion characteristics, such as stability and relative speed [56]. While the analytical basis for dynamic scaling and dynamic similarity can be drawn from classical mechanical and dynamical analyses, most investigations of dynamic similarity in the context of legged systems have focused on the observation of humans and animals [17,57–59]. While insightful, this focus limits the understanding of dynamic similarity since these studies are descriptive rather than predictive in nature. Recent efforts to extend the ideas of dynamic similarity to the design of scaled robotic systems have resulted in preliminary dynamic
scaling relations [16,60]. However, these relations include unnecessary assumptions that are holdovers from animal studies, resulting in more restrictive dynamic scaling relations than needed to produce a dynamically similar system.

### 2.3.1 Geometric, Kinematic, and Dynamic Similarity

Similarity is commonly used to compare models and prototypes of different scales [61, 62]. Though many forms of similarity may be considered, three classifications of similarity are essential to understand when developing scale prototypes: geometric, kinematic, and dynamic similarity. A model and prototype are said to be geometrically similar if they have the same shape [63]. This has two primary implications. First, all lengths must be scaled by a constant factor, $\alpha_L$, and second, all angles must be consistent between the model and prototype. If both of these conditions hold, then geometric similarity is said to be preserved.

A more stringent form of similarity is kinematic similarity. While geometric similarity requires the model and prototype to have the same shape, it places no restriction on the relative motion of the two objects. Kinematic similarity extends geometric similarity to additionally require that the velocity at any corresponding points between the model and prototype be scaled and have the same direction [63]. Since length must already be scaled by a constant factor, $\alpha_L$, adding an additional constant scale factor to time, $\alpha_T$, will insure that velocities are similar between the objects. Additionally, since angles must remain the same (carried over from geometric similarity), the velocity vectors for the model and prototype must be parallel at every location on the two and at every point in time.

The final core similarity type is dynamic similarity. While kinematic similarity will result in similar steady state motions for both model and prototype, it does not ensure the two to have similar transient responses to external stimuli. To incorporate this, dynamic similarity can be specified, which additionally requires the ratio of all forces to be constant and two be scaled by a constant factor, $\alpha_F$, between the model and prototype [63]. By preserving dynamic similarity, it can be assured that the steady-state and transient response of the scaled system will exhibit the same scaled and non-dimensional characteristics as the prototype [56].
2.3.2 Other Types of Similarity

The three similarity classifications addressed in Section 2.3.1 are the most commonly used and accepted methods to describe the similarity of objects, their motions, and their forces. However, other types of similarity have been outlined that are worth considering from a design standpoint.

The first is elastic similarity, which proposes a non-unity scaling relation between transverse and axial lengths. Elastic similarity [64] originates from consideration of buckling and bending of beams when under a load. Specifically, it hypothesizes that when scaling the axial length \( l \), the following relation should be used to preserve a constant strain in a beam:

\[
l = c \left( \frac{E}{\rho} \right)^{1/3} d^{2/3}
\]

where \( c \) is a constant that depends on the shape and angle of the beam, \( E \) is the elastic modulus of the beam, \( \rho \) is the density of the beam, and \( d \) is the transverse dimension. When developing this hypothesis, McMahon limited his consideration to biological systems that, in general, are composed of the same materials; this implied that \( E \) and \( \rho \) should both remain constant. Furthermore, since \( c \) is a constant dependent on the overall system shape, it would be invariant to scale changes as well. This led to the conclusion that the axial length should scale with the 2/3 power of transverse length.

This relation would imply that not all lengths should be scaled by the same, constant factor in order to preserve structural integrity and preserve the scaling of deflections. While at first glance, it may seem entirely contradictory to geometric scaling, this is not necessarily the case. Though geometric scaling in the most strict sense would not be possible, we can still effectively use both scaling procedures if we separate the lengths crucial to the dynamics from those crucial to strength and compliance. In a running system, the axial length of limbs will likely play an important role in the dynamics; however, the transverse length of the limbs probably does not, though it can greatly affect the strength. This allows the designer to scale different components in different ways while still maintaining the desired dynamic performance. Additionally, the reduction of (2.4) to \( l \propto d^{2/3} \) relies on the elastic modulus and density of the limb to remain constant with changing scale. However, in robotic design, the roboticist may use a range of materials to change these parameters as well to change the relationship between length and diameter.
An alternative similarity type is stress similarity. Proposed by Biewener [65], this hypothesis was drawn from observations of peak bone stress in animals, which was found to be roughly independent of size. This is inconsistent with geometric scaling, which predicts stress to scale $\propto M^{1/3}$ [66]. The method put forth by which animals are able to maintain stress similarity is to scale the effective mechanical advantage of their limbs $\propto M^{0.26}$, resulting in a progression from sprawled postures in small animals to upright postures in large animals.

In principle, two different implications can be drawn from the stress similarity hypothesis. The first is that stress should be independent of size, and to do so, the effective mechanical advantage should be altered. This implication is not as significant in the design of robots since, as addressed when discussing elastic similarity, a roboticist has the freedom to choose from a range of materials. Thus, if a higher stress is expected, a stronger material may be utilized. The second implication is that as size increases, the rate at which the required force increases can be slowed by increasing the effective mechanical advantage. This deviation from standard dynamic similarity can significantly decrease the actuator requirements and improve the efficiency and operating range of a robot.

### 2.4 Characterization of Robotic Performance

Once a new platform has been developed, it is often desirable, and even necessary, to quantify the system’s performance, whether to provide benchmarks for the expected behavior or to compare the robot to other platforms and models. In principle, robot performance can be characterized in a number of ways. The characteristics most appropriate may vary from application to application, but tend to gravitate towards measures of speed, efficiency, and robustness.

Speed is straightforward to ascertain and can be calculated as the time taken to travel a given distance. Some enhanced measures of speed have also been examined, such as relative speeds (e.g. bodylengths per second) or non-dimensionalized speeds (e.g. Froude number, $v^2/gl$) [13]. However, even with these slightly more complex formulations, the meaning of speed is well understood and can be quantified without significant difficulty. Efficiency can pose more of a challenge to both assess and comprehend, though not significantly so.
Efficiency can be established via a number of means, including power consumption, power per kilogram, and specific resistance \[67,68\], among others. As with speed, there are certain caveats that may complicate the discussion of power, but in general it is well understood.

A third key characteristic of mobile robotic performance is robustness. While speed and efficiency can be readily defined and quantified, there is an ambiguity to this performance measure. Depending on the context, robustness can refer to a number of different things, but can often be broken down into two separate considerations: mechanical robustness and stability. Mechanical robustness refers to how well the platform can withstand wear-and-tear and severe impacts without damage to the physical system (e.g. the legged platform can be picked back up and run again after falling without altering the behavior). Stability, on the other hand, is still open to a variety of interpretations. The traditional interpretation is the ability of the robot to stay at equilibrium when faced with a perturbation. For platforms with classical mobility mechanisms or quasi-static legged systems, this definition makes sense, as at any point, their is typically a base of support that would allow the robot to stand perfectly still and remain in that position even if it received a disturbance. However, for dynamical legged platforms, this definition is flawed as the ‘equilibrium’ behavior of the system is not truly steady-state but is instead a limit cycle behavior. Thus, all such platforms would be inherently classified as unstable even though they exhibit attractor behavior towards the limit cycle characteristic of a stable system.

It is clear that dynamical legged robots are stable in the sense that they are able to remain upright and continue to run effectively even over rough terrain that would hinder or incapacitate wheeled, tracked, or quasi-static legged systems. Thus an approach to quantify stability in a meaningful way for these platforms is required. Dynamic stability, or alternatively, disturbance rejection, is a measure of how well a dynamic system is able to return to an equilibrium state, or a limit cycle in the case of systems with a periodic orbit, following a perturbation to the system. In practice, dynamic stability can and should be characterized in two fundamentally different methods: 1) the maximum recoverable disturbance and 2) the rate of recovery from a disturbance \[69\]. In locomotion studies, both of these measures are important to ascertain a system’s ability to continue running and maintaining optimal performance as terrain or locomotion characteristics vary.
To begin to delve into these interpretations of stability, it is helpful to consider the system as a funnel, with the equilibrium or limit cycle situated at the lowest point [70]. In this sense, it is evident that for a state in which the system is set inside the ‘funnel’, it will trend towards the limit cycle. Within this context, a clear description of these two ways for characterizing stability can be defined. The maximum recoverable disturbance describes the largest perturbation for which the system remains within the ‘funnel’ and can thus recover to the nominal limit cycle. In locomotion studies, this measure of disturbance rejection is especially important when particularly large obstacles or perturbations are expected. However, most methods of calculating the maximum recoverable disturbance provide an indication of the largest perturbation that can be rejected once the system has settled to its periodic orbit. Thus, if the period between successive disturbances is smaller than the time it takes the system to recover to the limit cycle, the maximum recoverable disturbance by itself is not sufficient to ensure that the system will not fall. This motivates the use of the second measure, the rate of recovery from a disturbance.

While the maximum recoverable disturbance gives an indication of which states will recover to the limit cycle, the rate of recovery reflects how quickly a given system will return to its limit cycle following a perturbation. For the ‘funnel’ example above, this would be a measure of the slope rather than the width of the mouth. This measure is particularly relevant when disturbances are expected at a high frequency, since it will give an indication of whether successive disturbances will move the system farther away from its periodic orbit or whether the system will recover rapidly enough that it will stay in the neighborhood of the limit cycle. Aside from determining how quickly the system can reject disturbances, this measure is also important when the system itself is expected to undergo changes, either by changing control schemes or by altering its own configuration.

One example of this use is in sequential composition or ‘funneling’ of gaits [71] as depicted in Fig. 2.4. As animals run at increased speeds they typically undergo a series of transitions from walking to trotting to galloping. In a similar manner, legged robotic systems’ behaviors are planned as a composition of discrete dynamic behaviors. The RHex robot, for example, must transition from a slow walk into a jogging gait before it can utilize its highest speed gait [72]. These transitions are planned so that the limit cycle of each
Figure 2.4: Conceptual mapping of a gait composition for a running robot and illustrating the concept of gait funneling. The shaded regions represent the basin of attraction for the various gaits. The overlapping regions show at which states the robot is able to transition between two gaits.

A gait falls within the basin of attraction (described in Section 4.1.1) of the following gaits to which it may transition. Thus, when transitioning between gaits, the robot must wait until it is close to its limit cycle for a gait before it can attempt to transition to another one. In this way, the rate of recovery is important, since a rapid recovery from a disturbance is equivalent to a rapid recovery to the limit cycle following a gait transition. If the rate of recovery is slow, it takes longer to transition to the final desired gait, which may impact the effectiveness of the robot at performing its task.

### 2.5 Summary

The previous sections provided a review of many of the topics relevant to this dissertation. First, a discussion of biologically-inspired models commenced in which the motivation for their usage was addressed as well as a brief introduction to several dynamical, legged templates, including the SLIP and LLS models for running and the FG model for climbing. Next, a number of platforms that have been based on these models were considered, and an overview of several systems that have pressed the bounds of multi-modal locomo-
tion was presented. This was followed by an examination of scaling and preservation of similar locomotion, one of the primary challenges for developing robots from biologically-inspired models. Finally, a discussion of the issues addressing performance characterization commenced, with a particular focus on quantifying stability for dynamical systems.

The next few chapters will address how the work in this dissertation has built on these previous studies to provide an improved framework for the development of dynamical, legged systems capable of multi-modal locomotion.
CHAPTER 3
DYNAMIC SCALING OF DYNAMICAL SYSTEMS

As described in Section 2.3, scaling poses a significant challenge to the development of platforms at a size different from the motivating models or robots. However, this obstacle can be mitigated by the effective use of dynamic scaling. The following sections address the derivation of a set of dynamic scaling relations generally applicable to dynamic systems and the consideration of these scaling laws with particular regard to dynamical legged locomotion. Section 3.1 begins with an intuitive approach to deriving a set of scaling relationships for the system parameters of several reduced-order locomotion models, followed by a more direct method that can be generally applied to scale any arbitrary system parameter of a dynamic system. This is followed by a consideration of several constraints on dynamic scaling which are applicable to legged locomotion and a discussion of the theoretical insights afforded by these relationships.

Section 3.2 follows with simulation studies on the effects of the scaling relationships to verify their preservation of dynamic similarity with changes in scale. Each of the models considered address the effects of the various facets of the scaling relationships and aforementioned constraints as they relate to legged locomotion. This includes consideration of conservative locomotion without gravity (via the LLS model), conservative locomotion with gravity (via the SLIP model) and actuated, nonconservative locomotion (via the ncSLIP model).

In Section 3.3, the dynamic scaling approach is extended to experimental platforms. First, an investigation of two families of dynamic legged systems, RHex and DynoClimber, is undertaken to examine whether the various platforms within each family exhibit dynamic
similarity and to consider whether improved performance might be expected by certain platform modifications to more closely follow the proposed dynamic scaling relations. Second, an alternative usage of dynamic scaling, preserving dynamically similar locomotion when faced with varying payloads, is considered experimentally.

Section 3.4 considers several of the implications of dynamic scaling as they relate to design choices for scaled systems. This includes actuator selection, material properties and considerations for dynamic scaling in gravitationally-altered environments. Section 3.5 follows with some conclusions from this investigation.

### 3.1 Dynamic Scaling Relations

While scaling relations with the intent of preserving dynamic similarity have been previously derived [16], these examples rely on assumptions that need not be made and are overly restrictive for general dynamical systems. The following derivations demonstrate two ways by which a more generally applicable set of dynamic scaling relations can be found, the first being an intuitive approach and the other more suitable for determining the appropriate scaling relationships for arbitrary system parameters. We will use both of these methods to determine the scaling relations for several parameters relevant to reduce-order dynamical models\(^1\), which are listed in Table 3.1.

\(^1\)These parameters are those which are in particular relevant for the SLIP, LLS and FG models presented in Section 2.1. Note that while not every parameter is included (e.g. vertical and horizontal hip offset), these parameters are scaled in a manner analogous to others included in Table 3.1.
3.1.1 Force-Balance Approach

An intuitive approach by which to determine a viable set of dynamic scaling relations is to consider two arbitrary systems which are assumed to be dynamically similar and determining the appropriate scaling relationships that preserve the conditions of dynamic similarity, namely, that lengths, times and forces all scale by constant factor. To begin, the two systems can be described as

\[ \frac{m}{d^{2}x}{dt^{2}} = f \]  

and

\[ M \frac{d^{2}X}{dT^{2}} = F. \]  

Since this systems are dynamically similar,

\[ \alpha_{L} = X/x, \quad \alpha_{T} = T/t \quad \text{and} \quad \alpha_{F} = F/f. \]  

Substituting the scaling relations from (3.3) back into (3.2) yields

\[ M \frac{\alpha_{L}}{\alpha_{T}^{2}} \frac{d^{2}x}{dt^{2}} = \alpha_{F}f. \]  

By relating (3.1) and (3.4), we can find an equation to determine mass scaling for dynamic similarity:

\[ \alpha_{m} = M/m = \frac{\alpha_{F}\alpha_{T}^{2}}{\alpha_{L}}. \]  

The scale factor for the linear stiffness coefficient follows by assuming only elastic forces are acting on the two systems, governed by

\[ f = -kx \]  

and

\[ F = -KX, \]  

which, after substituting the scale factors from (3.3), becomes

\[ \alpha_{F}f = -K\alpha_{L}x. \]  

Relating (3.6) and (3.8) results in the stiffness scaling factor:

\[ \alpha_{k} = K/k = \frac{\alpha_{F}}{\alpha_{L}}. \]
The scale factor for system frequencies can be found by solving for the natural frequencies of the two elastically driven systems described above. The frequency of the first system is simply

\[ \omega = \frac{1}{2\pi} \sqrt{\frac{k}{m}} \]  

(3.10)

while the frequency of the second system, after the scale factors have been applied, is

\[ \Omega = \frac{1}{2\pi} \sqrt{\frac{\alpha_k k}{\alpha_m m}} = \frac{1}{2\pi \alpha_T} \sqrt{\frac{k}{m}}. \]  

(3.11)

A quick examination of (3.10) and (3.11) yields the equation for frequency scaling:

\[ \alpha \omega = \Omega / \omega = \frac{1}{\alpha_T}. \]  

(3.12)

A relation for the velocity scale factor can be found simply from

\[ x = vt \]  

(3.13)

and

\[ \alpha_L x = V \alpha_T t, \]  

(3.14)

which results in an expression of the scale factor of velocity as

\[ \alpha_v = V / v = \frac{\alpha_L}{\alpha_T}. \]  

(3.15)

We can then find the scale factor for damping by letting the forcing term act as a viscous damper:

\[ f = -bv \]  

(3.16)

and

\[ F = -BV, \]  

(3.17)

which can be rewritten as

\[ \alpha_F f = -B \alpha_v v = -B \left( \frac{\alpha_L}{\alpha_T} \right) v \]  

(3.18)

after substituting the scale factors in. Combining (3.16) and (3.18) yields the equation for the damping scale factor,

\[ \alpha_b = B / b = \frac{\alpha_F \alpha_T}{\alpha_L}. \]  

(3.19)
Table 3.2: Scale factors for System Variables of Dynamical, Legged Models

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Scale Factor</th>
<th>General Case</th>
<th>Constraint 1†</th>
<th>Constraint 2‡</th>
<th>Constraints 1† and 2‡</th>
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<td>$\alpha_L$</td>
<td>$\alpha_L$</td>
<td>$\alpha_L$</td>
<td>$\alpha_L$</td>
</tr>
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<td>$\alpha_L^3\alpha_T^{-2}$</td>
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<tr>
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<td>$\alpha_F\alpha_L^{-1}$</td>
<td>$\alpha_L^3\alpha_T^{-2}$</td>
<td>$\alpha_L^2$</td>
</tr>
<tr>
<td>Damping</td>
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<td>$\alpha_L^3\alpha_T^{-1}$</td>
<td>$\alpha_L^{5/2}$</td>
</tr>
<tr>
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<td>$1$</td>
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<td>Touch-Down Velocity</td>
<td>$\alpha_v$</td>
<td>$\alpha_L\alpha_T^{-1}$</td>
<td>$\alpha_L^{1/2}$</td>
<td>$\alpha_L\alpha_T^{-1}$</td>
<td>$\alpha_L^{1/2}$</td>
</tr>
<tr>
<td>Touch-Down Heading</td>
<td>$\alpha_\delta$</td>
<td>$1$</td>
<td>$1$</td>
<td>$1$</td>
<td>$1$</td>
</tr>
<tr>
<td>Touch-Down Leg Angle</td>
<td>$\alpha_\beta$</td>
<td>$1$</td>
<td>$1$</td>
<td>$1$</td>
<td>$1$</td>
</tr>
</tbody>
</table>

† Constraint 1: Gravity is scale independent ($\alpha_L = \alpha_T^2$).
‡ Constraint 2: Density is constant ($\alpha_m = \alpha_L^3$).
* The gravity scale factor is equivalent to the acceleration scale factor, though it is not specifically addressed herein.
The final scale factor which is non-unitary by definition is gravity (touch-down heading angle and touch-down leg angle are both invariant to scale since geometric similarity preserves all angles). A scaling factor can be found by letting gravity act as the forcing term on both systems:

\[ f = mg \]  
(3.20)

and

\[ F = MG, \]  
(3.21)

which is rewritten as

\[ \alpha_F f = \alpha_m mG = \frac{\alpha_F \alpha_T^2}{\alpha_L} mG \]  
(3.22)

after substitution of (3.3). Substituting (3.20) into (3.22) results in a gravity scaling factor (or equivalently, acceleration scaling factor) of

\[ \alpha_g = G/g = \frac{\alpha_L}{\alpha_T^2}. \]  
(3.23)

The resulting scaling relationship are compiled and listed in the third column of Table 3.2.

### 3.1.2 Dimensional Analysis Approach

While the approach presented in Section 3.1.1 provides an intuitive means of determining the scale factors of various parameters that will preserve dynamic similarity, it is cumbersome when faced with determining a scale factor for an arbitrary system parameter. An alternative method is to use dimensional analysis. This approach to deriving the dynamic scaling laws follows from the Buckingham-Pi Theorem [73], by which the system variables can be written in terms of several fundamental physical quantities. While this method is often used to produce a non-dimensionalized set of variables, we take a different approach, using the fundamental physical quantities to produce scaling factors for all the other system variables. In this case, we use length, time, and force as the fundamental quantities, following the conditions proposed by Alexander to produce dynamic similarity [57].

According to the dynamic similarity hypothesis, if each of the fundamental dimensions is scaled by a constant factor, \( \alpha_L \), \( \alpha_T \), and \( \alpha_F \), the scaled system will be dynamically similar to the original. For the purpose of this discussion, we assume that \( \alpha_L \) is the primary scaling
parameter while $\alpha_T$ and $\alpha_F$ are free parameters. Using the dimensional analysis approach and the computed dimensions of the various system parameters (see the third column of Table 3.1), the scaling factors for each parameter can be computed. Furthermore, when comparing these results to the force-balance approach, we see that the scale factors for the parameters of dynamical legged locomotion are the same when found by either method. This supports the hypothesis that by preserving these scaling relationships in the design of legged locomotors, dynamically similar behavior can be expected by the resulting systems.

### 3.1.3 Constraints on Dynamic Scaling

A careful examination of the relationships derived in the previous two sections shows a potential flaw in this set of dynamic scaling laws. When scaling gravity, it is evident from (3.23) that the gravitational acceleration will not remain independent of scale for arbitrary scale factors $\alpha_L$ and $\alpha_T$. To remedy this conundrum, we introduce the constraint that gravity must be scale independent, resulting in an equation for $\alpha_T$ in terms of $\alpha_L$:

$$\alpha_g = 1 = \alpha_L \alpha_T^{-2}$$

$$\alpha_T = \alpha_L^{1/2}. \quad (3.24)$$

Substituting (3.24) for $\alpha_T$ in the general scaling relations results in a new set of scaling relations, shown in the fourth column of Table 3.2. With this constraint, dynamic similarity can be maintained during scaling with a free scaling parameter, $\alpha_F$. Note that this constraint is only necessary in situations for which gravity acts on the system. For those in which gravity does not have an effect (e.g. the Lateral Leg Spring model), the general scaling laws may be applied, leaving both $\alpha_T$ and $\alpha_F$ as free scaling parameters.

A second, often applied constraint for dynamic scaling is that $\alpha_m = \alpha_L^3$. This constraint follows from strict adherence to geometric similarity, in addition to the assumption of a constant density for the system. While the constant density assumption may have merit for biological creatures since many animals are composed of the same biological tissues (e.g. bone, muscle, fat, etc.), which are present in roughly similar proportions [57], there is no need for such an assumption in robotic design due to the flexibility in structural materials. Additionally, strict geometric scaling may not always be the proper approach in the design of robotic structures, as discussed in Section 2.3.2. Since the internal structure may be
scaled in a different manner than the overall body, the effective density may change even when the same structural materials are used. However, for thoroughness and to show how this discussion relates to previously derived scaling laws [16], we examine the effects of this constraint on the scale factors, which results in an equation for $\alpha_F$:

$$\alpha_m = \frac{\alpha_F \alpha_T^2}{\alpha_L} = \alpha_L^3$$

$$\alpha_F = \frac{\alpha_L^4}{\alpha_T^2}. \tag{3.25}$$

The fifth column of Table 3.2 shows scale factors for the system variables when only the constant density assumption is made, while the sixth column shows the scale factors when both constraints from (3.24) and (3.25) are applied. The scale factors when applying both constraints results in the same scale factors that had previously been reported in [16].

### 3.1.4 Insights from Dynamic Scaling Derivation

In addition to the practical application of the general and constrained scaling relations shown in Table 3.2, several insights can be gleaned by considering the implications of these findings. The first, and most significant from a design standpoint, is the availability of a free parameter for developing a scaled version of a system. This provides considerable flexibility when compared with the previous scaling relationships, which restricted dynamically similar platforms to utilize a specific parameter set at the desired scale. Furthermore, if the gravity constraint can be relax, for instance, in LLS-like systems, an additional free parameter is available to the designer.

Second, due to $\alpha_F$ being independent of the other fundamental scaling factors, we can build off of previous insights regarding the scaling of system power requirements [16]. Earlier scaling relations showed that the required specific power for the system increases proportional to the square root of length ($\alpha_p = \alpha_L^{1/2}$), which leads to power limitations as the desired scale increases since actuator specific power is relatively independent of size. Thus, specific power can become a major limiting factor when increasing the scale of a system. However, this is not as restrictive if $\alpha_F$ is a free parameter, as will be addressed in greater detail in Section 3.4.1.

The third insight from this derivation is that even with the freedom afforded by the free scaling factor, the nominal speed of a scaled system is still uniquely prescribed when the
gravity constraint must be considered since $\alpha_F$ has no effect on velocity scaling. Though this may be a limiting factor for design, it also is particularly relevant to the discussion of dynamic similarity. In particular, it means that there is a specific speed that will preserve dynamic similarity of the original system when it is scaled to a particular size. Note that this only relates to the gravitationally constrained case of dynamic scaling; if gravity does not affect the system, this requirement does not hold and the speed of a locomotion can be arbitrarily set at any length scale.

3.2 Application of Scaling to Locomotion Models

To verify the success of these scaling relations in preserving dynamic similarity, we examine the effects of scaling on a two reduced-order models of legged locomotion, the Lateral Leg Spring model and the Spring Loaded Inverted Pendulum model. Since dynamic scaling should only alter those parameters which can be derived from fundamental parameters, non-dimensional output parameters should remain invariant for dynamically scaled systems [74]. A list of the non-dimensional output parameters chosen to verify the dynamic similarity laws is contained in Table 3.3. In addition to the non-dimensional output parameters, the dimensional touch-down velocity magnitude, peak ground reaction force and peak spring strain energy are included\(^2\).

3.2.1 Horizontal Plane Locomotion

The first model we examine is a point-mass variant of the Lateral Leg Spring (pmLLS) model. The LLS model describes the horizontal plane trajectory of the center of mass while running and has been demonstrated to effectively capture the dynamics of small running animals, such as geckos and cockroaches [56], as reviewed in Section 2.1.2. Since only horizontal plane motion and forces affect this model, it allows for the verification of the general dynamic scaling relations (see the third column of Table 3.2 without considering the effect of gravity.

\(^2\lambda_{\text{max}}\) is found from the numerically computed Jacobian at the touch-down Poincaré section. $N_{Fv}$ and $N_{Str}$ are defined as $v^2/\omega l$ and $u/\omega l$, respectively. $R_V$ and $R_F$ are the ratios of the peak lateral or vertical velocity or force to the peak fore-aft velocity or force. $\delta$, $v$, $F_{\text{max}}$ and $E_{\text{max}}$ are measured directly from the simulation.
Table 3.3: Output Parameters for Verification of Dynamic Similarity

<table>
<thead>
<tr>
<th>Output Parameters</th>
<th>Symbol</th>
</tr>
</thead>
<tbody>
<tr>
<td>Maximum Eigenvalue</td>
<td>$\lambda_{\text{max}}$</td>
</tr>
<tr>
<td>Froude Number</td>
<td>$N_{Fr}$</td>
</tr>
<tr>
<td>Strouhal Number</td>
<td>$N_{Str}$</td>
</tr>
<tr>
<td>Peak Velocity Ratio</td>
<td>$R_{V}$</td>
</tr>
<tr>
<td>Peak Force Ratio</td>
<td>$R_{F}$</td>
</tr>
<tr>
<td>Touch-Down Heading Angle</td>
<td>$\delta$</td>
</tr>
<tr>
<td>Touch-Down Velocity Magnitude</td>
<td>$v$</td>
</tr>
<tr>
<td>Peak Ground Reaction Force</td>
<td>$F_{\text{max}}$</td>
</tr>
<tr>
<td>Peak Spring Strain Energy</td>
<td>$E_{\text{max}}$</td>
</tr>
</tbody>
</table>

Table 3.4: Output Parameters for Scaled LLS with General Dynamic Scaling Relations

<table>
<thead>
<tr>
<th>$(\alpha_L, \alpha_T, \alpha_F)$</th>
<th>(1,1,1)</th>
<th>(2,1,1)</th>
<th>(1,2,1)</th>
<th>(1,1,2)</th>
<th>(2,3,5)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\lambda_{\text{max}}$</td>
<td>0.222</td>
<td>0.222</td>
<td>0.222</td>
<td>0.222</td>
<td>0.222</td>
</tr>
<tr>
<td>$N_{Fr}$</td>
<td>2.780</td>
<td>5.600</td>
<td>0.700</td>
<td>2.780</td>
<td>0.622</td>
</tr>
<tr>
<td>$N_{Str}$</td>
<td>2.828</td>
<td>2.828</td>
<td>2.828</td>
<td>2.828</td>
<td>2.828</td>
</tr>
<tr>
<td>$R_{V}$</td>
<td>0.104</td>
<td>0.104</td>
<td>0.104</td>
<td>0.104</td>
<td>0.104</td>
</tr>
<tr>
<td>$R_{F}$</td>
<td>3.069</td>
<td>3.069</td>
<td>3.069</td>
<td>3.069</td>
<td>3.069</td>
</tr>
<tr>
<td>$\delta$</td>
<td>0.103</td>
<td>0.103</td>
<td>0.103</td>
<td>0.103</td>
<td>0.103</td>
</tr>
<tr>
<td>$v$</td>
<td>0.700</td>
<td>1.400</td>
<td>0.350</td>
<td>0.700</td>
<td>0.467</td>
</tr>
<tr>
<td>$F_{\text{max}}$</td>
<td>0.017</td>
<td>0.017</td>
<td>0.017</td>
<td>0.034</td>
<td>0.087</td>
</tr>
<tr>
<td>$E_{\text{max}}$</td>
<td>5.06e-5</td>
<td>1.01e-4</td>
<td>5.06e-5</td>
<td>1.01e-4</td>
<td>5.06e-4</td>
</tr>
</tbody>
</table>

The pmLLS model is formulated in the same manner as the canonical LLS model with the body momentum of inertia $I$ and the leg offsets $d_1$ and $d_2$ set to zero. A numerical simulation of this model was implemented in MATLAB® [75]. The nominal model parameter values were taken from Schmitt and Holmes [54]. We examine how the system behaved when the fundamental scale factors, $\alpha_L$, $\alpha_T$, and $\alpha_F$, are arbitrarily altered and the physical variables (see Table 3.1) are modified according to the general scaling relations. Simulations were run for the nominal system and for four scaled systems. For each scaled system, we found the fixed point and calculated the non-dimensional output parameters, which are shown in Table 3.4.

These results show that all non-dimensional output parameters aside from $N_{Fr}$ remain constant for arbitrary choices of the fundamental scale factors. However, since gravity does
not affect this system, $N_{Fr}$ is not well defined and variation can be expected.\(^3\) While the non-dimensional outputs remained constant, the dimensional outputs clearly fluctuate with scale. Thus, for this system, dynamic similarity is preserved for any choice of fundamental scale factors as long as the general dynamic scaling relations are applied.

### 3.2.2 Sagittal Plane Locomotion

While the scale effects on the pmLLS model demonstrate that the general scaling relations can be used to produce dynamically similar locomotion, most running robots and animals need to take gravity into account to some extent, even when moving in a way characteristic of pmLLS locomotion. For example, in sprawled posture animals, while locomotive forces are largely in the horizontal plane, support is needed to keep the body elevated off the ground [76]. To provide us with a lens to look at the scaling laws when under the effect of gravity, we examine two variants of a sagittal plane locomotion model.

**Conservative SLIP Model.** The first sagittal plane model considered is the conservative Spring Loaded Inverted Pendulum (SLIP) model. This model has a similar set of system parameters to the pmLLS model, so it serves as a simple analog to examine whether the addition of gravity, by itself, and the subsequent use of the gravity constrained scaling relations will generate dynamically similar motions.

A numerical simulation of the SLIP model was implemented in MATLAB using the same framework developed for the LLS model. The nominal parameter values were selected from Ghigliazza, et al. [77]. Two sets of experiments were performed. In the first, we verified that for a gravitationally affected system like SLIP, $\alpha_F$ could be independently varied while maintaining dynamic similarity. The nominal system output parameters were compared to those when an arbitrary non-unity $\alpha_F$ was selected while the other fundamental scale factors were held constant, the results of which are shown in Table 3.5. These results demonstrate that the non-dimensional output parameters are independent of the force scaling factor. This is in agreement with the predicted behavior (see column 3 of Table 3.2), since the addition of the gravity constraint does not limit $\alpha_F$. Furthermore, while $F_{max}$ and $E_{max}$ fluctuate with force scaling, $v$ remains invariant to changes in $\alpha_F$.

\(^3\) $N_{Fr}$ is often used to classify scale-independent gaits, but since it’s definition includes gravity, it is not expected to apply to a gravity-independent runner. In essence, the gravity parameter could be arbitrarily set to 0, making $N_{Fr}$ infinite for any system not influenced by gravity.
Table 3.5: Output Parameters for Scaled SLIP when Varying $\alpha_F$

<table>
<thead>
<tr>
<th>$(\alpha_L, \alpha_T, \alpha_F)$</th>
<th>(1,1,1)</th>
<th>(1,1,2)</th>
<th>(1,1,5)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\lambda_{max}$</td>
<td>0.288</td>
<td>0.288</td>
<td>0.288</td>
</tr>
<tr>
<td>$N_{Fr}$</td>
<td>4.129</td>
<td>4.129</td>
<td>4.129</td>
</tr>
<tr>
<td>$N_{Str}$</td>
<td>22.922</td>
<td>22.922</td>
<td>22.922</td>
</tr>
<tr>
<td>$R_V$</td>
<td>0.156</td>
<td>0.156</td>
<td>0.156</td>
</tr>
<tr>
<td>$R_F$</td>
<td>3.117</td>
<td>3.117</td>
<td>3.117</td>
</tr>
<tr>
<td>$\delta$</td>
<td>0.141</td>
<td>0.141</td>
<td>0.141</td>
</tr>
<tr>
<td>$v$</td>
<td>8.000</td>
<td>8.000</td>
<td>8.000</td>
</tr>
<tr>
<td>$F_{max}$</td>
<td>33.005</td>
<td>66.010</td>
<td>165.026</td>
</tr>
<tr>
<td>$E_{max}$</td>
<td>5.447</td>
<td>10.893</td>
<td>27.233</td>
</tr>
</tbody>
</table>

The second set of experiments verified the relationship between $\alpha_L$ and $\alpha_T$ as predicted by (3.24). A series of simulations were run in which $\alpha_L$ and $\alpha_T$ were varied independently, from 0.1 to 10 and 0.316 to 3.16, respectively. The output parameters were computed for each trial and the results for a selection of them are shown in Fig. 3.1. The plots for the other output parameters are omitted since they show the same overall behavior as the parameters in Fig. 3.1. Examining each of the three plots, we see that the non-dimensional output parameters remain constant along a line with a slope of 0.5. These results agree with effect on the output parameters predicted by the dynamic scaling relations. As derived in (3.24), it is expected that $\alpha_L$ must be equal to $\alpha_T^2$ to maintain dynamic similarity when a system is influenced by gravity. Since these are log-log plots, this corresponds to a squared relationship between the two scale factors, in agreement with our previously derived results. Thus, we can confirm that to maintain dynamic similarity in scaled systems under the effects of gravity, $\alpha_L = \alpha_T^2$.

It is worth noting that the rate at which the output parameters change when not moving along the constant slope (i.e. transversely along the diagonal band in Fig. 3.1) is different between parameters. This reflects the fact that certain parameters are more sensitive to variation from dynamic similarity than others. This could be useful in choosing a set of non-dimensional parameters to use in confirming the dynamic similarity of a prototype platform to a previous model or robot, as the consistency of more sensitive parameters would more strongly indicate dynamic similarity.
Non-conservative SLIP Model. While the SLIP model provides insight into the use of gravitationally constrained dynamic scaling, it neglects several key components that must be considered for real-world applications. The first of these is system losses. While theoretical models often assume that systems are loss-less, the losses present in all real-world systems have a significant impact. The second missing component of the conservative model is a control approach that, at the most basic level, provides a means of recuperating energy lost to dissipative elements. While dynamically scaling the physical structure may enable a scaled platform to move in a dynamically similar fashion to its unscaled counterpart, if the controller is not properly adjusted for the new scale, the performance of the scaled platform will not be dynamically similar. While the controller may be re-optimized for the new scale, we hypothesize that dynamically scaling the controller, in addition to the physical platform, will generate dynamically similar motion without the need for controller re-optimization.

To test this hypothesis and examine the effects of dynamic scaling on a lossy system, we investigate an implementation of the non-conservative Spring Loaded Inverted Pendulum (ncSLIP) model. This model is similar to the SLIP model with the addition of a viscous damper with a damping coefficient of $b$ in parallel to the leg spring (see Fig. 2.1) and a controller to modulate system energy and maintain a stable gait. The controller used is the Active Energy Removal (AER) controller, proposed by Schmitt [78], which has since
been demonstrated to stabilize sagittal plane hopping in both simulation [79] and physical investigations [80]. This controller has two components. The first is a feed-forward, energy modulation scheme that removes and adds energy during stance, resulting in a periodic gait that is robust to ground height variation. The second uses feedback to adjust the touch-down angle $\beta$, which results in faster recovery from perturbations as well as recovery from a wider range of perturbations [80]. Since three of the four parameters used in this controller are non-dimensional, only one parameter, the actuation timing parameter, needs to be scaled\(^4\).

The ncSLIP model was implemented in the same framework as the previous SLIP and LLS models and the same experiments were run as for the SLIP model. The results for varying $\alpha_F$ while holding the other fundamental scaling factors constant are shown in Table 3.6. As with varying $\alpha_F$ in the SLIP model, these results indicate that the force scaling factor can be varied independently of the other two fundamental scaling factors while maintaining dynamic similarity between the original and scaled simulations even when damping and energy incorporation are present in the system.

The results for independently varying $\alpha_L$ and $\alpha_T$ on the ncSLIP model are shown in Fig. 3.2. As with the SLIP results in Fig. 3.1, the non-dimensional outputs remain constant along any line with a slope of 0.5 on the plots, corresponding to a quadratic relationship between $\alpha_L$ and $\alpha_T$.

\(^4\)Since this parameter has the dimensions of $T$, it scales proportionally to $\alpha_T$. 

<table>
<thead>
<tr>
<th>$(\alpha_L, \alpha_T, \alpha_F)$</th>
<th>(1,1,1)</th>
<th>(1,1,2)</th>
<th>(1,1,5)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\lambda_{\text{max}}$</td>
<td>0.212</td>
<td>0.212</td>
<td>0.212</td>
</tr>
<tr>
<td>$N_{Fr}$</td>
<td>3.748</td>
<td>3.748</td>
<td>3.748</td>
</tr>
<tr>
<td>$N_{Str}$</td>
<td>18.244</td>
<td>18.244</td>
<td>18.244</td>
</tr>
<tr>
<td>$R_V$</td>
<td>0.357</td>
<td>0.357</td>
<td>0.357</td>
</tr>
<tr>
<td>$R_F$</td>
<td>3.527</td>
<td>3.527</td>
<td>3.527</td>
</tr>
<tr>
<td>$\delta$</td>
<td>0.311</td>
<td>0.311</td>
<td>0.311</td>
</tr>
<tr>
<td>$v$</td>
<td>7.939</td>
<td>7.939</td>
<td>7.939</td>
</tr>
<tr>
<td>$F_{\text{max}}$</td>
<td>50.235</td>
<td>100.470</td>
<td>251.175</td>
</tr>
<tr>
<td>$E_{\text{max}}$</td>
<td>12.618</td>
<td>25.235</td>
<td>63.089</td>
</tr>
</tbody>
</table>
3.2.3 Discussion of Simulation Results

For the pmLLS case, it can be seen that all three of the fundamental scaling parameters can be scaled independently and all of the output parameters other than $N_{Fr}$ remain invariant. As $N_{Fr}$ is not well-defined for this system and, consequently, can be ignored, these results indicate that for any system in which gravity does not apply, the designer of a dynamically scaled robotic system has free-reign to select any set of fundamental parameters $\alpha_L$, $\alpha_T$, and $\alpha_F$ in the design process when using the general set of dynamic scaling relations.

As expected for the sagittal plane models, gravity provides one additional constraint (which is introduced since $N_{Fr}$ must now be preserved), limiting the system to two free scaling parameters, one of which is used to pick the size of the scaled model. This leaves one free parameter, $\alpha_F$, which plays a significant role because it is closely related to the mass of the platform. Under the previously developed dynamic scaling relations, $\alpha_F \propto \alpha_L^3$. However, this assumption need only be made if the density of the original and scaled models are the same. In biology, as previously mentioned, this is typically the case, since animals have roughly similar material compositions. However, for robots, numerous materials are available. This provides a roboticist with flexibility in that they can design the scaled platform with any mass (within bounds, which will be discussed in Section 3.4.1) as long as...
as the model stiffness, the only other parameter affected by $\alpha_F^5$ can be adapted to the required value. This holds a particular interest for tasks requiring a legged robot to retrieve a payload. By incorporating a method by which leg stiffness could be adapted during operation, such a platform would still be able to move in a dynamically similar fashion after it had increased its mass by the addition of the payload.

Studying the variation of the fundamental scaling factors for the ncSLIP model results in the same conclusions as for the SLIP model, though both damping and stiffness need to be adapted with changes in mass. Furthermore, the actuation methods examined preserve dynamic similarity when appropriately scaled themselves. However, actuator limits need to be considered to determine whether specific actuators can provide sufficient power to cope with increasing scale.

### 3.3 Dynamic Scaling of Dynamical Legged Robots

Given the results of the simulation studies in Section 3.2, it is expected that the dynamic scaling relationships presented in Table 3.2 can be directly applied to dynamical legged platforms to preserve locomotion characteristics at various scales. To investigate this, we will consider two families of dynamical robots for which platforms of different sizes have been developed and examine whether dynamically similar behavior is seen between them. Furthermore, we will examine the application dynamic scaling to field applications, specifically insofar as $\alpha_F$ can be used to adjust leg stiffness to maintain consistent performance when a payload is added or removed from a platform.

#### 3.3.1 Scaling of the RHex and DynoClimber Families

We begin by considering two distinct families of dynamical legged platforms: RHex, which was designed for running on rugged terrains that are near-level, and Dynoclimber, which was built to run up vertical surfaces. Several versions of each of these platforms have been developed with varying degrees of success, with RHex robots ranging in size from $1kg$ to $50kg^6$ and Dynoclimber robots ranging from $200g$ to $2.5kg$. Physical and locomotion

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5For the SLIP model, since the damping coefficient is zero, it will be invariant to scale even though the scale factor is non-zero.

6Unsuccessful efforts to develop an even larger version have been attempted, including a $200kg$ system on the television series, Prototype This! [81].
Table 3.7: System Parameters for the RHex Family

<table>
<thead>
<tr>
<th>Parameter</th>
<th>RHex</th>
<th>Scaled to Edubot*</th>
<th>Edubot*</th>
</tr>
</thead>
<tbody>
<tr>
<td>Body Length (m)</td>
<td>0.51†</td>
<td>0.37</td>
<td>0.36</td>
</tr>
<tr>
<td>Leg Length (m)</td>
<td>0.16†</td>
<td>0.115</td>
<td>0.115</td>
</tr>
<tr>
<td>Mass (kg)</td>
<td>7.5†</td>
<td>3.2</td>
<td>3.2</td>
</tr>
<tr>
<td>Leg Stiffness (Nm⁻¹)</td>
<td>1900†</td>
<td>1100</td>
<td>960</td>
</tr>
<tr>
<td>Speed (ms⁻¹)</td>
<td>2.7‡</td>
<td>2.29</td>
<td>2.25</td>
</tr>
<tr>
<td>Stride Frequency (s⁻¹)</td>
<td>4.2</td>
<td>4.95</td>
<td>5.1</td>
</tr>
<tr>
<td>Froude Number (−)</td>
<td>4.67</td>
<td>4.67</td>
<td>4.49</td>
</tr>
<tr>
<td>Strouhal Number (−)</td>
<td>4.02</td>
<td>4.02</td>
<td>3.86</td>
</tr>
<tr>
<td>Specific Resistance (−)</td>
<td>0.84†</td>
<td>0.84</td>
<td>0.51</td>
</tr>
</tbody>
</table>

† Moore, et al. [82]
‡ Weingarten, et al. [72]
* Scaled using the gravitationally constrained scaling laws (Table 3.2, Column 4) with the following fundamental scaling factors: α_L = 0.719 and α_F = 0.457.
* Galloway, et al. [83]

characteristics of two successful RHex platforms, the original RHex robot [82] and the smaller Edubot [83], are shown in Table 3.7, while those of two DynoClimber platforms, the original DynoClimber [16] and the smaller BOB [33], are shown in Table 3.8.

Though many similarities exist between the RHex and Edubot platforms (including electronics, body design, and leg type), the physical design and controller parameters for each platform was optimized separately. Both optimizations were performed using a Nelder-Mead routine with a cost function of SR/v^2, which weighted gaits to favor speed and efficiency. Despite the fact that dynamic scaling was not explicitly considered in the design process, there are remarkable similarities in the resulting performance characteristics, as shown in Table 3.7. It is also worth noting the primary difference, namely the improved specific resistance, with Edubot is primarily due to the reduction in leg stiffness after a leg compliance optimization study [83], which had not been performed on the original RHex platform. This suggests that reducing the leg stiffness of the original RHex platform from 1900Nm⁻¹ to 1550Nm⁻¹ would improve the efficiency at its 7.5kg scale.

While these and other RHex-like platforms at a similar scale have performed well [84,85], attempts at producing significantly larger and smaller versions have not enjoyed the same success. Larger platforms, including a 50kg version, have failed to move at speeds faster
than a walk and even then have exhibited limited stability and robustness compared to the original hexapedal platform. A primary factor for the decline in performance with increased size is that limited consideration was made for the proper tuning of leg stiffness\(^7\). In fact, leg stiffness for these platforms has been significantly higher than dynamic scaling calls, which tends to result in both decreases in speed and stability [86]. A further contributing factor is that system power, which will be discussed in Section 3.4.1, was not scaled properly to provide the requisite power for dynamically scaled running. On the other hand, the comparatively poor performance of platforms smaller than 1\(kg\) is likely a result of mechanical complexity at a small scale, which leads to inefficiencies and lack of mechanical robustness.

While currently developed oversized (> 30\(kg\)) and undersized (< 2\(kg\)) versions of the RHex platform have been unable to reproduce the success of the original, future scaled versions that take dynamic scaling into consideration may be able to reproduced the expected scaled behavior. For larger systems, the primary considerations are related to system power and leg stiffness. While reducing the overall weight would go a long way towards limiting the system power requirements, this is not always practical. An alternative that may prove more viable is to increase the weight budget allocated to actuator and power storage components while keeping the platform weight consistent, increasing the power available for consumption while limiting the increase in system power requirements. Additionally, consideration of novel materials for the legs or use of alternative configurations may allow for the requisite stiffness while still allowing for enough deflection. For small platforms, simplification of the structural design would improve the performance and use of compliant and monolithic structures and design processes such as Shape Deposition Manufacturing (SDM) [87] or Smart Composite Microstructure (SCM) fabrication [88] rather than individual parts and fasteners could increase the reliability.

While the similarity of the dynamic behaviors for the original RHex and Edubot platforms suggest that dynamic scaling will produce analogous behaviors to optimization for running, these results do not necessarily extend to other dynamic, legged modalities. This

\(^7\)Rather than attempting to match the dynamically scaled stiffness, attempts at larger versions of RHex have used stiffer legs that result in less deflection during normal operation. This is partially due to the lack of available materials that possess both the requisite stiffness and are able to sustain large enough deflections without failure.
led to an examination of scaled versions of the DynoClimber family. These dynamical climbing platforms do not show quite the same level of dynamic similarity as the RHhex family even though preliminary dynamic scaling was considered to some extent in the design process, as can be seen in Table 3.8. This is partially because of the limited use of optimization with this family.\(^8\)

A number of additional factors likely contribute to the discrepancies between the expected behaviors at the different scales. First and foremost, the actuation schemes utilized of the two platforms are decidedly different, with DynoClimber using a self-exciting controller to achieve maximum speeds while BOB utilized a single actuator to drive both legs, which were rigidly fixed 180° out of phase by its transmission.\(^9\) The difference in the control approaches is also indicative of one of the trade-offs inherent to changes in scale. Due to the limited physical dimensions and payload capacity for smaller platforms, simpler and less powerful electronics are available, restricting the complexity of potential control algorithms. On the other hand, at smaller scales, actuator power density increases relative to the required system power (see Section 3.4.1), which makes relatively stronger actuators available.

Another set of factors that likely reduced the dynamic similarity of the two dynamical climbers are the surface properties of the climbing substrate. Both DynoClimber and BOB ran on a vertical surface covered in a similar Berber carpet; thus, the foot-surface contact properties remained similar rather than scaling with the dynamic scaling relationships. Certain properties in particular would have had a significant effect. First, while the FG template models the foot-surface contact as a rigid joint, the compliance of the carpet is not negligible, particularly at larger sizes. Second, because the attachment mechanism for these climbers uses a dactyl to grab the loops in the carpet, the stroke length is shortened by some amount as the foot establishes a hold on the surface. Since both of these platforms used a similar means to attach to the surface, the stroke length would have been shortened by a constant amount, rather than following the dynamic scaling laws, further influencing

---

\(^8\)The controller for DynoClimber was hand-tuned to achieve peak performance, while the controller for BOB has no tuning, per se, as it is strictly controlled by the applied voltage.

\(^9\)Additionally, the top speeds for BOB were achieved by increasing the actuator power by raising the applied voltage, which is not a tenable means for improved performance during prolonged operation.
Table 3.8: System Parameters for the DynoClimber Family

<table>
<thead>
<tr>
<th>Parameter</th>
<th>DynoClimber†</th>
<th>Scaled to BOB‡</th>
<th>BOB*</th>
</tr>
</thead>
<tbody>
<tr>
<td>Body Length (m)</td>
<td>0.4</td>
<td>0.15</td>
<td>0.18</td>
</tr>
<tr>
<td>Stroke Length (m)</td>
<td>0.13</td>
<td>0.05</td>
<td>0.05</td>
</tr>
<tr>
<td>Mass (kg)</td>
<td>2.6</td>
<td>0.187</td>
<td>0.187</td>
</tr>
<tr>
<td>Leg Stiffness (Nm(^{-1}))</td>
<td>640</td>
<td>119</td>
<td>141</td>
</tr>
<tr>
<td>Speed (ms(^{-1}))</td>
<td>0.67</td>
<td>0.42</td>
<td>0.35</td>
</tr>
<tr>
<td>Stride Frequency (s(^{-1}))</td>
<td>3.5</td>
<td>5.6</td>
<td>6.5</td>
</tr>
<tr>
<td>Froude Number (–)</td>
<td>0.35</td>
<td>0.35</td>
<td>0.25</td>
</tr>
<tr>
<td>Strouhal Number (–)</td>
<td>1.47</td>
<td>1.47</td>
<td>1.08</td>
</tr>
</tbody>
</table>

† Lynch, et al. [16]
‡ Scaled using the gravitationally constrained scaling laws (Table 3.2, Column 4) with the following fundamental scaling factors: \(\alpha_L = 0.385\) and \(\alpha_F = 0.0719\).
* Dickson, et al. [33]

The degree of similarity. These factors illustrate that the contributing elements to dynamic similarity are not limited to the platform itself, but can also be affected by external factors.

These platforms demonstrate the utility of dynamic scaling in experimental design. Had dynamic scaling been applied to the Edubot platform, the extensive redesign and optimization should not have been required. Furthermore, the similarity between DynoClimber and BOB shows that even when utilizing vastly different drive systems and controllers, the performance is fairly similar. It is also worth noting that the discrepancy in the DynoClimber platforms is larger than for the RHex platforms in large part due to the differences in drive train and controller, which was not a factor for the RHex platforms.

### 3.4 Implications of Scaling for Robotic Design

The dynamic scaling relations from Section 3.1 and verification thereof in Section 3.2 show that from a theoretical standpoint, platforms can be developed at any scale by the appropriate selection of fundamental scale factors. However, in reality, additional factors establish some limits on the design of such scaled systems and provide other considerations that must be accounted for. While previous work has briefly addressed actuator scaling and materials strength [89], we will extend this discussion given the conclusions of our analytical
work. Additionally, we will consider the adaptation of the dynamic scaling relations to preserve locomotion behaviors in gravitationally-altered environments.

3.4.1 Actuator Scaling

Consideration of actuator scaling first requires an understanding of how power requirements scale for a given system. Though many complexities could be included, such as inefficiencies and basal power consumption, for a first-order approximation, the mechanical power required can serve as an adequate surrogate for the system power requirements. Mechanical power is defined as

\[ p = f v, \]  

where \( p \) is the mechanical power, \( f \) is the force exerted, and \( v \) is the velocity of the system. This equation can be used to define a power scaling factor as

\[ \alpha_p = \frac{P}{p} = \frac{\alpha_F \alpha_L}{\alpha_T} = \alpha_F \alpha_L^{1/2}. \]  

(3.27)

There are two immediate implications that can be drawn from (3.27). First, for power to remain invariant to scale, \( \alpha_F = \alpha_L^{-1/2} \) (or equivalently \( \alpha_L = \alpha_F^{-2} \)). While possible, this is often an impractical restriction since the mass of the platform, proportional to \( \alpha_F \), would have to decrease as the length scale of the platform increases.

Furthermore, there is typically no need for power to be invariant to scale; the power supplied by the actuators simply needs to meet the scaled system’s power requirements. This can be more appropriately considered by the scaling of specific power, the ratio of power to weight. Since the scale factor for specific power of a system can be formulated as \( \alpha_p/\alpha_m \), it is evident that specific power scales as \( \alpha_L^{1/2} \) and is thus independent of \( \alpha_F \). For platforms of reduced scale, this is a boon and often affords small robots a surplus of available power. However, for platforms at increased scales, the specific power scaling can pose a problem since, in general, the specific power of various actuators is scale independent. At first glance, this negates any flexibility afforded by the free scaling parameter. However, there are two ways in which this flexibility is still present. First, while \( \alpha_F \) specifies the scaling of the overall system mass, individual components need not scale in the same way. Thus, if the
Table 3.9: Scale Considerations for Common Actuators

<table>
<thead>
<tr>
<th>Actuator Type</th>
<th>Specific Power</th>
<th>Maximum Efficiency</th>
<th>Common Scale</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hydraulic†</td>
<td>$10^3 W/kg$</td>
<td>98%</td>
<td>&gt; 50 kg</td>
</tr>
<tr>
<td>Pneumatic†</td>
<td>$10^4 W/kg$</td>
<td>40%</td>
<td>500 g – 50 kg</td>
</tr>
<tr>
<td>Electromechanical‡</td>
<td>$10^2 W/kg$</td>
<td>92%</td>
<td>10 g – 50 kg</td>
</tr>
<tr>
<td>Smart Materials†</td>
<td>$10^3 W/kg$</td>
<td>99%</td>
<td>&lt; 10 g</td>
</tr>
</tbody>
</table>

† Adapted from Huber, et al. [90] and Zupan, et al. [91]
‡ Adapted from Maxon Motor AG [92]

Proportion of the weight budget allocated to actuation is increased, the actuators’ specific power can be raised accordingly.\(^{10}\)

Second, since $\alpha_F$ is independent of scale, the mass of a platform can be increased without increasing the specific power requirements. Thus, if a greater specific power is needed, alternative actuators with improved specific power can be utilized even if they are heavier. To examine this in greater detail, we must consider actuators that may be viable candidates. A number of different actuator types can be utilized for supplying power to a robotic system. Among the most common are hydraulic, pneumatic, electromechanical, and smart materials actuators. And, as would be expected, all of these are best suited for particular applications and scales. Table 3.9 provides a summary of the typical power availability and scale for these actuators.

Hydraulic actuators generate forces and torques from the pressure built up in a hydraulic fluid. These actuators are well suited for use on large systems, since they are capable of high efficiency and specific power operation. Though they are particularly effective on large platforms, their utility is limited in small scale application. This is results from the challenge of reducing weight at small scales due to the need for a pump, a reservoir, and sufficient tubing and piping and the complexity of incorporating all of these components.

Pneumatic actuators provide an alternative that operates on a similar principle. However, pneumatic devices are often smaller than hydraulic ones. While these actuators still provide a high specific power, they are restricted by the compressibility of air, which limits peak stresses and complicates control. However, pneumatics are better suited for small scale

\(^{10}\)This procedure must be used judiciously, as increasing the actuator weight budget will correspondingly restrict the structural and payload weight budgets
applications since they do not require a dense, hydraulic fluid. Furthermore, a compressor can be used to replace a reservoir, though this limits the power and efficiency further.

Electromechanical actuators can provide an alternative to pressure-based devices. While they cannot achieve the same specific power as hydraulic or pneumatic devices, their ease of use can make up for it. They are also very simple and straightforward to control and typically provide a much higher degree of accuracy than the aforementioned actuators. Furthermore, the widespread use of electromechanical actuators has resulted in their affordability and availability at a wide range of scales, particularly at small sizes. However, at very small scales, the efficiency of such actuators drops precipitously.

Once systems at the micro- or nano-scales are being considered, a new class of actuators must be examined. A number of smart materials approaches, including piezoelectric stacks, shape memory alloys, and dielectric elastomers have been developed. Depending on the application and actuator requirements, these options can provide a high specific power and efficiency; however, they are greatly limited in size, which has restricted their use in larger systems. Cost is also a severe deterrent when other, conventional actuators are available and practical.

3.4.2 Materials Scaling

The presence of $\alpha_F$ as a free scaling parameter allows dynamic similarity to be maintained at a given size for a range of masses. This can be used on one hand to carry larger loads, or on the other to reduce platform weight, which in turn reduces the system power requirements. However, the platform weight can only be reduced so much before structural weakness decreases mechanical robustness and results in failure of the support structure. As discussed in Section 2.3.2, stresses scale proportionally to $M^{1/3}$ when geometric similarity is preserved at a constant density (though deviation from strict dynamic scaling can mitigate this to an extent). Thus as scale increases, larger structures and/or stronger support elements are required to keep the structure from braking. However, at small scales, slight structures made of lighter, weaker materials can provide sufficient strength to keep the robot intact.

As a first-order approximation of structural failure, we can consider the stress in a beam under a load, $\sigma = \frac{M y}{I}$, where $\sigma$ is the normal stress on the beam, $M$ is the moment produced
by the load, \( y \) is the distance of a test point from the neutral axis, and \( I \) is the moment of inertia of the beam cross-section. From this equation, it is clear that to maintain a stress when loads and lengths increase, the cross-sectional moment of inertia must be increased proportionally, which is at odds with the allocation of additional weight budget to the actuators. However, with the availability of various engineering materials, an alternative approach is to allow stresses to increase, but choose an appropriate material that will be able to withstand the higher stresses. Thus for smaller platforms, the use of weaker, lighter materials such as cardboard (\( \sim 10 \text{MPa} \) yield strength) and ABS plastic (\( \sim 45 \text{MPa} \) yield strength) will likely suffice, while larger platforms may require stronger materials such as aluminum (\( \sim 125 \text{MPa} \) yield strength) or steel (\( \sim 700 \text{MPa} \)). In addition, choosing beam cross-sections with a greater moment of inertia relative to density (e.g. I-beams instead of square beams) will improve the load bearing capacity relative to the weight of the structure.

3.4.3 Scaling in Extraterrestrial Applications

In the previous sections, we have examined how dynamic scaling should be approached when gravity is invariant (either zero or non-zero) but not addressed what considerations should be made when the magnitude of the gravitational acceleration changes. In this case, gravity is still invariant to scale since the size of the robot does not affect it; rather, the local environment does. This is of particular relevance for extra-terrestrial applications in which the original platform is likely to be tested on earth before being deployed in a different gravitational environment. Thus, scaling laws need to be adapted to account for the fact that there may be an external gravitational scaling factor, \( \alpha_g \).

For this situation, we examine the dynamic scaling laws and adapt them to allow for dynamically similar locomotion of a platform when the scale is kept constant (i.e. both length and mass are unchanged) while the gravitational acceleration changes. We start by looking at the expression for the gravitational scale factor in the general case (third column of Table 3.2 or (3.23)). Since length is assumed to remain unchanged, \( \alpha_L = 1 \) and a relation between \( \alpha_g \) and \( \alpha_T \) can be expressed as

\[
\alpha_T = \sqrt{\frac{1}{\alpha_g}}.
\] (3.28)
Table 3.10: Scale Factors for Non-Unity $\alpha_g$

<table>
<thead>
<tr>
<th>Scale Factors</th>
<th>Scaling Expressions</th>
<th>Scaling for Mars$^\dagger$</th>
<th>Scaling for the Moon$^\dagger$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha_l$</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>$\alpha_m$</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>$\alpha_k$</td>
<td>$\alpha_g$</td>
<td>0.378</td>
<td>0.165</td>
</tr>
<tr>
<td>$\alpha_b$</td>
<td>$\alpha_g^{1/2}$</td>
<td>0.615</td>
<td>0.407</td>
</tr>
<tr>
<td>$\alpha_\omega$</td>
<td>$\alpha_g^{1/2}$</td>
<td>0.615</td>
<td>0.407</td>
</tr>
<tr>
<td>$\alpha_v$</td>
<td>$\alpha_g^{1/2}$</td>
<td>0.615</td>
<td>0.407</td>
</tr>
<tr>
<td>$\alpha_\delta$</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>$\alpha_\beta$</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

$^\dagger$ Note that the gravitational acceleration on Earth, Mars and the Moon were set to 9.807$ms^{-2}$, 3.711$ms^{-2}$ and 1.622$ms^{-2}$, respectively.

We can then determine the effect of $\alpha_g$ on $\alpha_F$ from the mass scale factor, which also is equal to 1:

$$\alpha_m = \frac{\alpha_F \alpha_T^2}{\alpha_L} \Rightarrow 1 = \alpha_F \alpha_T^2$$

$$\alpha_F = \frac{1}{\alpha_T^2} = \alpha_g.$$  \hfill (3.29)

With these expressions for the three fundamental scale factors, the scale factors for the system variables can be determined, as shown in the second column of Table 3.10. Additionally, the third and fourth columns show the values for the various scale factors that should be applied if scaling from the Earth’s gravity to Mars’ or the Moon’s, respectively. From these relations, it is clear that dynamic similarity is achievable when moving a robot to a new gravitational field without requiring modifications to the size or mass of the system. Rather, by altering leg stiffness and damping, in addition to frequency and velocity, dynamically similar motions can be generated.

### 3.5 Conclusions

In this chapter, a thorough examination of dynamic scaling and its implications have been discussed. We began by using a first-principles approach to derive a set of dynamic scaling relations that should preserve dynamic similarity for any dynamic system. An alternative derivation based on the Buckingham-Pi Theorem was presented as a faster and
more straightforward method of determining the dynamic scaling relations for any arbitrary parameter of a dynamic system. We then compared the relations obtained from both of these approaches with previous dynamic scaling relationships to understand the causes of the differences in the scaling laws. This resulted in the determination of two constraints that had been implicitly applied to previous scaling work, the invariance of both gravity and density with scale. While the gravity invariance constraint is applicable to most dynamical running platforms, density invariance need not be preserved, and by relaxing this constraint, a free scaling factor $\alpha_F$ is made available, which can be used to mitigate scale limitations that arise from actuator or materials considerations.

Using the newly obtained dynamic scaling relationships, we considered the preservation of dynamic similarity on dynamical, legged models of locomotion. In this study, three disparate models of locomotion were considered at a variety of scales specified by arbitrary selection of the three fundamental scale factors. We found that when gravity was not a factor, all three fundamental scale factors could be chosen independently and as long as the general dynamic scaling relationships were followed, dynamic similarity would be preserved. Furthermore if gravity was an influential factor, a power relationship existed between $\alpha_L$ and $\alpha_T$ when dynamic similarity was preserved that matched the constraint imposed by the gravity constraint. Finally, we confirmed that it is not only necessary to scale physical properties, but control parameters as well following the dynamic scaling relationships.

Given these insights, we considered two families of dynamical, legged robots with platforms at various scales. For each family, the scaled system parameters were compared to those suggested by the dynamic scaling relations to see how closely parameters selected via optimization, a much more time and resource-intensive process, matched. Furthermore, the performance of the platforms was analyzed to see whether dynamic similarity was preserved and to investigate if improved performance could be expected with a closer adherence to dynamic similarity. We found that, for the RHex family in particular, the parameters chosen via optimization matched closely to those predicted by dynamic scaling. Furthermore, the consistency of non-dimensional output parameters suggested that the scaled platforms showed a high degree of dynamic similarity. An examination of why platforms with greater changes in scale tended to demonstrate reduced levels of performance was also undertaken,
leading to the identification of several design suggestions for improved adherence to dy-
namic similarity at various scales as well as the identification of the need to understand the
actuator and material constraints on scaling.

This led to a discussion of the impact of scaling on how both actuators and materi-
als scale. For actuation, we identified that power tends to increase with both length and
force ($3.27$), such that specific power increases with $\alpha_L^{1/2}$. This provides a challenge in
terms of actuator scaling, since by first-order approximation actuators’ specific power tends
to be invariant to scale. This suggests that with changes in size, it is necessary to consider
alternative forms of actuation as well as to reallocate relative actuator mass to accommo-
date the need for more power. For materials as well, the stress within structural elements
increases with size, which in turn requires larger beams to withstand the increased forces.
However, by the proper selection of materials and the use of beam structures with a high
moment of inertia relative to mass, it is possible to design larger support structures that a
light enough to allow for dynamic scaling to be applied.

The final consideration of this chapter was the application of dynamic scaling to allow
for dynamically similar locomotion when faced with a different gravitational acceleration,
such as for extraterrestrial exploration. We showed that by adjusting the gravity scaling
constraint, dynamic similarity can be preserved by changing the leg stiffness and damping
as well as the time scaling factor $\alpha_T$. With these modifications, dynamic, legged platforms
designed in an Earth gravity setting should be able to perform in a predictable and optimal
fashion on other planetary bodies.
CHAPTER 4

QUANTIFICATION OF STABILITY ON EXPERIMENTAL LEGGED PLATFORMS

The following sections address the work performed in determining an efficient and efficacious means for quantifying disturbance rejection on experimental systems. Section 4.1 begins with a review of a number of metrics that have been previously considered in some capacity for measuring dynamic stability or disturbance rejection. In addition, the concepts of gait indicators and gait disturbances are addressed.

Sections 4.2 and 4.3 follow with a simulation study to consider the efficacy of the previously reviewed metrics. This is followed by an experimental investigation of the same metrics in Sections 4.4 and 4.5. The chapter concludes with a summary of the result and consideration of the study implications in Section 4.6.

4.1 Disturbance Rejection Metrics

In the subsections that follow, we briefly review several disturbance rejection metrics that could be employed in locomotion studies, including basin of attraction, maximum eigenvalue magnitude, gait sensitivity norm, decay ratio and settling time.

4.1.1 Basin of Attraction

The basin of attraction, \( \text{BoA} \), is the set of all states that asymptotically converge to a specific equilibrium state (note that for a single system, several basins may exist that converge to different equilibria). Since the basin of attraction includes all converging states, it can be used to quantify the maximum recoverable disturbance.
Figure 4.1: Conceptual drawings of various metrics of stability, from top to bottom: (A) The basin of attraction, $BoA$, and maximum eigenvalue magnitude, $\lambda_{\text{max}}$, of the return map, (B) the gait sensitivity norm, $GSN$ and decay ratio, $DR$, over the first $n$ steps, (C) the settling time, $TS$, and (D) the mean state variance, $MSV$. 

\[
GSN = \frac{1}{|\mathcal{X}|} \sum_{k=1}^{N} \sum_{j=0}^{n-1} (g_j(k) - g_m(k))^2 \\
DR = \frac{1}{n} \sum_{k=2}^{n} \frac{|g_k - g_m|}{|g_1 - g_m|^{1/2}}
\]
This concept can be illustrated by a damped ball and basin, as shown in Fig. 4.1A. In this example, a perturbation moves the ball from its equilibrium position to another location in the basin. Assuming that the ball is still at rest after the perturbation (e.g. it is placed at some location), all perturbations that yield a new ball location within the peaks of the basin will result in ball trajectories that asymptotically approach the equilibrium solution, while those outside of the peaks will diverge from this solution. Relaxing this assumption and allowing the perturbed ball to have a nonzero velocity as well, any perturbation for which the total energy of the ball remains less than the potential energy at the peaks of the basin will remain within the basin and will asymptotically converge back to the equilibrium state. The basin of attraction of the equilibrium solution is therefore composed of all states in which the ball is between the peaks and the energy of the ball is less than is required to reach the basin peaks.

While the theory behind the example above can be applied to any dynamical system, some considerations must be taken into account for systems which do not converge to an equilibrium state, but rather to a periodic orbit. For locomotion systems, which exhibit this behavior, a Poincaré section is typically employed. The Poincaré section is used to sample the dynamical system at a specific event, such as flight apex or leg touch-down. The dynamics of the continuous system instantiate a discrete recurrence mapping relating the system state at subsequent intersections with the Poincaré section. In this manner, the basin of attraction is defined by the set of all states on the Poincaré section that asymptotically converge to the fixed point of the resulting Poincaré map, where the fixed point is the intersection of the limit cycle with the Poincaré section. While employing this strategy reduces the computational complexity of determining the basin of attraction, it limits the types of perturbations those that are of relevance specifically at the Poincaré section. For example, if the Poincaré section is defined at leg touch-down, impulsive perturbations applied during the stance or flight phases could result in a fall before the next intersection with the Poincaré section.

A primary difficulty in employing the basin of attraction as an indication of the system’s capacity for disturbance rejection is that it can be difficult to compute, both numerically and experimentally. From a numerical perspective, computation of this metric becomes
intractable as the state dimension increases. Even systems with few state dimensions require performing simulations for a large number of steps to determine convergence to the limit cycle over a large grid of possible initial conditions. Experimental computations of this metric are even more cumbersome, since the same difficulties arise with regard to the state dimension. Furthermore, testing the physical system can be prohibitively challenging, as extensive testing of initial conditions may be difficult, and since the edges of the basin must be found, many falls will be encountered, which may result in physical damage. Another deficiency of the basin of attraction is that while it is well equipped to determine the maximum recoverable disturbance, it says nothing about the rate at which a system recovers from a disturbance.

4.1.2 Maximum Eigenvalue Magnitude

While the basin of attraction characterizes the maximum recoverable disturbance, the rate at which the system recovers from perturbations to the system is more aptly described by the maximum eigenvalue magnitude. This metric quantifies the rate at which small perturbations are dissipated.

For the ball and basin example in Fig. 4.1A, this recovery rate is quantified as the slope of the basin local to the equilibrium state. The maximum eigenvalue magnitude has been utilized to quantify local disturbance rejection for many reduced order modeling studies [8, 10, 77, 93–95] as it is computationally simple to determine in simulation. Calculation of this metric requires linearizing the Poincaré map about the fixed point. The Jacobian of the linearized Poincaré map is then found, from which the eigenvalues are computed. The magnitude of the maximum eigenvalue is then taken as a measure of recovery rate. The maximum eigenvalue is used because the corresponding eigenvector is the slowest to converge and is, therefore, the limiting rate governing the system’s return to the limit cycle.

While this metric is computationally tractable and provides a simple indication of whether minute perturbations applied at the limit cycle will grow or decay, a primary deficiency of this metric is the range over which it is valid. Because it relies upon a linearization of the nonlinear system, the predicted recovery rate is only valid in a small region about the fixed point. While a basin of attraction can be determined from a local Lyapunov function created from the linearization about the equilibrium solution, this basin of attraction is
only a conservative estimate of the true basin of attraction for the full nonlinear system. Moreover, significant external perturbations can move the state beyond the region of convergence for which the local linearization holds. As such, behavior of the nonlinear system in response to large perturbations can vary greatly from the predictions of the maximum eigenvalue magnitude of the linearized system. From an experimental perspective, computing the maximum eigenvalue magnitude can also be difficult for a number of reasons. First, because the eigenvalues reflect the local behavior, extremely small perturbations must be applied, which requires highly sensitive and accurate measurement of all state variables and will be quite susceptible to both system and measurement noise. Second, the determination of the eigenvalues requires the states to be individually perturbed, which is a difficult and arduous task. However, recent efforts in determining eigenvalues experimentally have begun to make progress and may prove fruitful for future robotic studies [96].

4.1.3 Gait Sensitivity Norm

The gait sensitivity norm (GSN) [69] provides a measure of the dynamic response of one or more gait indicators in response to a disturbance. Described further in both Section 4.1.7 and [69], gait indicators are measurable system quantities whose variations can adequately capture failure modes of interest in response to external disturbances. As with eigenvalues, numerical computation of the gait sensitivity norm in simulation requires linearization about the equilibrium solution as well as calculation of sensitivity matrices that pertain to the variations in system states and gait indicators due to the disturbances, as given by

\[
\begin{align*}
x_{n+1} &= Ax_n + Be_n, \\
g_n &= Cx_n + De_n,
\end{align*}
\]  

where \(x_n\) represents the deviation of the system’s state at the Poincaré section of the \(n\)-th step from the fixed point, \(e_n\) represents the disturbance applied to the system (for example, a change in ground height), and \(g_n\) represents the value of the chosen gait indicator at the Poincaré section of the \(n\)-th step. The matrix \(A\) is the Jacobian matrix, which describes how the current state will affect the state at the following stride. The vectors \(B\) and \(C\) are the sensitivity of the system state to a disturbance and the effect of the state on the gait.
indicator, respectively. The scalar $D$ is the sensitivity of the gait indicator to a disturbance. For a chosen gait indicator, the gait sensitivity norm is computed from the $H_2$ norm as

$$\left\| \frac{\partial g}{\partial e} \right\|_2 = \sqrt{D^2 + \text{trace}(B^T PB)}$$

where $P$ is the solution to the discrete Lyapunov matrix equation

$$A^T PA - P + C^T C = 0.$$ (4.4)

The gait sensitivity norm represents the standard deviation of the system output response, represented by a gait indicator, to white noise or impulse inputs. As such, this metric is well suited for use in cases where the disturbances are represented by either single or multiple changes in height, characteristic of locomotion over rough terrain. A benefit of this formulation lies in the ability to select individual system characteristics that quantify locomotion performance and vary in response to disturbances that instigate failure modes, such as falling. The gait sensitivity norm has previously been shown to correlate well with the maximum recoverable disturbance, characterized by the maximal magnitude of Gaussian white noise for which the system did not fall, in both a walking model and a physical prototype [69]. More recently, a similar correlation has been shown for the conservative SLIP model as well [97]. While direct correlations with recovery rate were not explicitly computed, the gait sensitivity norm was also found to increase in magnitude as the distance from the limit cycle increased [69]. From a physical implementation perspective, the gait sensitivity norm has difficulties in dealing with significant amounts of noise relative to the size of the perturbation, and experimental computation requires a large number of steps after encountering a single disturbance.

### 4.1.4 Decay Ratio

The decay ratio (DR) quantifies disturbance rejection by characterizing the rate at which a particular gait indicator returns to its equilibrium value following a perturbation. A variant of this metric, with the gait indicator being the Euclidean distance from the equilibrium solution, was proposed by Su and Dingwell in examining passive-dynamic walking performance over a rough slope [98]. In this study, the original decay ratio formulation of [98] is
extended to an $n$-step decay ratio as

$$
\frac{1}{n} \left[ \sum_{k=2}^{n+1} \left( \frac{g_k - g_\infty}{g_1 - g_\infty} \right)^{\frac{1}{k-1}} \right]
$$

where $n$ denotes the total number of strides employed, $g_k$ denotes the gait indicator at touch-down, $k$ is the number of strides after the perturbation, and $g_\infty$ indicates the value of the gait indicator at equilibrium. Fig. 4.1B illustrates the use of apex height as a gait indicator to calculate the decay ratio using (4.5) following a drop-step.

The use of decay ratio to determine the rate of the recovery from a perturbation is potentially advantageous for several reasons. The decay ratio quantifies the recovery rate from a perturbation relative to the size of the initial perturbation. It does not rely upon a linearization and is, therefore, well suited for examining recovery from large perturbations that may move the system state out of the range for which the linearization is valid.

Furthermore, computation in either simulation or experiment only requires running the system for a few strides. Considering more than one stride captures the potential coupling between states and between the system and controller. This is particularly important when utilizing a single indicator. On the other hand, restricting the examination to the first few strides emphasizes the immediate response to large perturbations and reduces stride-to-stride noise. As with the gait sensitivity norm, a benefit of the decay ratio lies in its flexibility to be constructed from a variety of gait indicators. However, this metric does not provide any information regarding long term behavior, and it may fail to identify cases in which initial convergence is replaced by long-term divergence away from the limit cycle. As well, it is expected that the decay ratio will be sensitive to noise for small perturbations from the limit cycle.

4.1.5 Settling Time

While the settling time (TS) is often associated with the unit step response of a second order linear differential equation, this metric can also be applied in the analysis of the rate of recovery from disturbances. For example, it has been utilized to quantify the recovery rate of cockroaches when subjected to lateral perturbations [99]. The settling time provides an indication of the time required for the system to return to, and remain within a specified...
bound of, the limit cycle, as illustrated in Fig. 4.1C. In a similar fashion to the gait sensitivity norm and decay ratio formulations, the settling time can also be constructed to analyze performance with respect to specific gait indicators.

As the bound utilized for the settling time decreases, this metric becomes more sensitive to noise that might prevent the system from remaining within the specified distance from the limit cycle, though the increased sensitivity enables a higher resolution of the disturbance rejection behavior to be examined. Unlike the gait sensitivity norm and decay ratio, this metric does not provide any information regarding how much the state response deviates from the limit cycle solution due to the perturbation. Another caveat for determining settling time is that since this metric examines the convergence after a single step, successive perturbations, even minute ones, may impact the results, especially in systems which converge slowly or when the settling bounds are tight. While this is not a hindrance in simulation, it can necessitate the use of large bounds and low resolution in experimental usage.

4.1.6 Mean State Variance

In running over rough terrain, frequent ground perturbations will prevent the system from recovering the unperturbed periodic orbit. However, the distance that the system remains from the limit cycle during such a sequence of perturbations is certainly of interest, especially in regards to the sequencing of gaits [72] previously discussed. Consequently, we consider the mean state variance (MSV) as a measure of the variability of state variables when running over continuously varying terrain, as shown in Fig. 4.1D. It can be found by determining the average distance that the system remains away from the limit cycle when running over successive perturbations. Note that for perturbations that inherently alter the limit cycle (e.g. ground height, if body altitude is used as a state variable), the limit cycle to which the system would converge in the absence of further perturbations should be used to calculate distance from the limit cycle.

This metric is directly indicative of the rate of recovery from a perturbation. For systems which rapidly recover to the limit cycle (e.g. dead beat control), this metric will be approximately zero since there would be almost no variation in the state variables. However,
for systems that recover more slowly, the mean state variance will stay large since each successive perturbation will keep the system away from the limit cycle. Faster rates of recovery after a perturbation will result in smaller variance from the mean over the course of terrain traversal and vice-versa for gaits with slower rates of recovery. Furthermore, this metric can be computed in both simulation and experiment. The mean state variance handles both large and small perturbations and does not rely on a linearization of the system. It explicitly considers the response to a continuous sequence of perturbations. This approach most directly captures the disturbance rejection rate that is the focus of this study. An often prohibitive drawback of this approach is the necessity of having an experimental setup that allows for continuous application of controlled perturbations over a large number of steps. In addition, the system needs to be capable of measuring the full state of the system and be robust enough to function without degraded performance in face of inevitable falls through extended experimental trials. Additionally, a large number of steps and perturbations are typically required to ensure that the mean state variance remains statistically similar for different random terrain selections with the same mean step height.

4.1.7 Gait Indicators and Disturbances

Gait indicators are measurable quantities indicative of locomotion performance whose variations can adequately capture failure modes of interest in response to external disturbances. Disturbances may come in many forms and can have significantly differing impact on the system evolution. For running systems, common disturbances include variations in the ground height, surface stiffness, and friction between the ground and the foot. Impulsive forces are also potential disturbances, though they are not inherent to the running itself (e.g. another object impacting). For the purpose of this discussion, we are assuming that the disturbance of interest is ground height variation, since this is by far the most common perturbation to a running system. Thus, it is essential to ascertain which gait indicators are most closely correlated with the rate of recovery following height disturbances.

The selection of gait indicators is driven by several factors: their ability to adequately capture locomotion failures, their use in prior locomotion studies, and the ease of sensing/computing such quantities in a physical system. Several indicators were initially examined spanning from the state variables at various Poincaré sections, to timing and trajectory
measurements, to the system energy and work performed. Initial examination of the potential indicators allowed for many to be discarded due to several factors including, but not limited to: equivalence to other indicators, invariance to perturbations, or direct relation to control parameters. Selection of the gait indicators for use in this study was settled by choosing indicators that examined several dissimilar factors. This ensured that a range of quantities were examined on the system rather than potentially focusing on a subset which were not as sensitive to the recovery from a perturbation. The proposed indicators include: stance duration, distance of the apex state from the limit cycle, the system energy at touch-down, and the leg angle at lift-off. Each of these quantities can exhibit distinctive variations in the strides preceding a fall, thereby capturing the failure mode of interest. Of these quantities, both the stance duration and the system energy at touch-down were previously examined utilizing the gait sensitivity norm for small perturbations in ground height for a walking model and prototype [69]. The apex state has been utilized in prior studies in determining local stability as part of an apex return map [100, 101]. While the leg angle at lift-off has not been explicitly utilized in examining locomotion performance in response to perturbation, it plays an important role the Active Energy Removal controller implemented in this study.

From a sensing/computation perspective, some gait indicators require a little to no sensing above that which is already incorporated into many robot designs. For example, the leg angle at lift-off can be measured via an optical encoder while the stance duration can be measured using contact sensors and a timer. While these indicators are simple to compute in the single-legged hopper of this study, indicators specific to a particular leg may be more difficult to generalize for multi-pedal models and robots. Conversely, system indicators such as the system energy at touch-down or the deviation in the apex state from the limit cycle, while invariant to the number of legs, are more difficult to sense and compute experimentally and may necessitate the addition of sensory units not otherwise required on a platform. The performance of both leg and system gait indicators will be examined in this study.
4.1.8 Disturbance Rejection Metric Selection

In this study, the AER controller, previously developed in [102] is implemented in three different locomotive systems of varying complexity: a generalized SLIP model, a more physically realistic simulation that includes mechanical damping, a linkage-based transmission system and torque/speed motor limitations, and an experimental platform comprised of a planar, single-legged hopping robot.

While examination of both maximum recoverable disturbances and rates of recovery would provide insight into the disturbance rejection capabilities, the nature of the controller itself and its natural stability renders the analysis of the range of recoverable disturbances a moot point since the platform and corresponding simulation have been observed to fall solely due to physical limitations (i.e. not due to divergence from the limit cycle) for reasonably chosen control parameters. For example, at the speed ranges considered, when the robot or simulation encounters a step up, as long as the foot does not hit or scuff the step, the original gait is recovered. As a result, the maximum recoverable disturbance for a step up is simply the maximum ground clearance of the foot (or some variation thereof that takes into account that the step may be encountered when the foot is not maximally off the ground). Likewise, when encountering a step down, as long as the leg spring does not fully compress (i.e. bottom out), the system recovers back to the unperturbed periodic orbit. As such, these systems are better suited to investigate the rate of recovery from a disturbance since the maximum recoverable perturbation is limited by the physical constraints of the platform rather than the underlying dynamics.

The disturbance rejection metrics selected for examination in this study therefore include those that are capable of characterizing the rate of recovery from an external perturbation rather than the maximum recoverable disturbance. These include, as previously detailed, maximum eigenvalue magnitude, gait sensitivity norm, decay ratio and settling time. The basin of attraction or methods that can be used to estimate the basin of attraction, such as the maximum recoverable disturbance or the mean first passage time [103], are not examined in this study. The mean state variance is utilized as a characteristic measure of the recovery rate, since they are inherently linked, as described in Section 4.1.6. To
determine the applicability of the other metrics, the correlation of each metric to the mean state variance will be examined as the locomotion characteristics are varied.

4.2 Reduced-Order Running Models

To begin the investigation of quantifying disturbance rejection, two running simulations were considered, a generalized, conservative SLIP model and a robot-based, non-conservative SLIP model. These two models are described in the following sections.

4.2.1 The Generalized SLIP Model and Active Energy Removal

The generalized SLIP model investigated follows the dynamics of the SLIP model presented in (2.2) and (2.3). Due to the inability of this model to inject or dissipate energy, additional control mechanisms must be incorporated to allow for the recovery of a unique limit cycle when subjecting the model to energetic perturbations (e.g. ground height variation).

The ‘Active Energy Removal’ (AER) controller was developed to stabilize the SLIP model in response to energetic perturbations while capturing the higher-level leg function exhibited in animal locomotion. The controller design was inspired by recent animal locomotion studies on both guinea fowl and cockroaches. Specifically, locomotion studies with guinea fowl revealed that when running over an unexpected drop, the animal utilizes changes in leg length and leg angle at touch-down (termed posture-dependent leg actuation) to manage system energy during the next stride [104]. Additionally, studies of cockroaches running over a terrain comprised of obstacles up to three times the hip height distributed in a random array revealed that the insect could still run at 80% of its unobstructed speed [105]. Furthermore, it was observed that even for steps in which a leg did not contact the ground (i.e. due to a significant drop), leg progression was not stopped to search for a foothold but rather continued along a similar trajectory as if it had encountered the ground, indicating that a feed-forward trajectory was likely prescribed for the leg.

From these results, Schmitt [102] hypothesized a control strategy for use with the original SLIP model and examined, via simulation, its ability to enable the model to recover from energetic perturbations. For the AER controller, the desired leg angle at touch-down for
the upcoming step is determined via feedback while the leg length actuation is specified in a feed-forward manner. Simulations demonstrated that this control strategy enables recovery from terrain drops of up to 40% of the hip height, similar to the performance seen in guinea fowl [104].

The leg actuation strategy utilized to modulate the system energy during stance varies the force-free leg length $\zeta_0$ in a feed-forward fashion as:

$$\zeta_0 = l_{\text{nom}} - l_{\text{dev}} \sin (\omega_{\text{des}} t), \quad (4.6)$$

where $l_{\text{nom}}$ represents the nominal leg length, $l_{\text{dev}}$ is the amplitude by which the rest length is varied from the nominal leg length, and $\omega_{\text{des}}$ is the actuator driving frequency used to control the rate of energy modulation for leg actuation. In this formulation, $t$ denotes the elapsed time from the beginning of the current stance phase, such that $t = 0$ and $\zeta_0 = l_{\text{nom}}$ at the beginning of each stance (i.e. when touch-down occurs).

In this actuated formulation, changes in the force-free leg length during stance roughly result in energy removal during leg compression and energy addition during leg extension. For a conservative, periodic gait, gait symmetry in $\zeta$ and $\dot{\zeta}$ about mid-stance ensures that the energy absorbed during the first half of the stance phase equals that added during the latter half, such that the energy at lift-off equals that at touchdown. However, such gait symmetry is destroyed in the presence of external perturbations or when the system is away from the limit cycle. In these instances, the leg lift-off event can occur earlier or later than would be expected with the periodic gait, thereby directly affecting the amount of energy added back into the system during the extension phase. This allows the AER controller to return the system back to the limit cycle, since if the system has more energy than the periodic orbit, lift-off will occur before the leg has extended back to the nominal rest length resulting in an energy loss. Likewise, if the system has less energy than the periodic orbit, the stance phase will last longer and the leg will extend past the nominal rest length, increasing the energy in the system.

In addition to the leg actuation protocol in (4.6), an adaptive leg touch-down angle control law was implemented and is defined as:

$$\beta_{n+1}^{TD} = \beta_n^{LO} + c (\beta_n^{TD} - \beta_{\text{des}}^{TD}) , \quad (4.7)$$
where \( c \) is a dimensionless control parameter and \( \beta_{des}^{TPD} \) is a control parameter which sets the desired touch-down angle when a periodic gait is achieved. This control law was developed to improve the stability of the velocity heading angle at touch-down and has been shown to stabilize most gaits via appropriate choice of \( c \) [78].

### 4.2.2 Robot-Based SLIP Model

As with the generalized SLIP model, the robot-based simulation dynamics are based on (2.2) and (2.3). However, two additional terms were added to the stance dynamics formulation to incorporate rotational and linear damping. This produced a stronger correlation between the model dynamics and those of the single-legged hopping robot used for the experimental investigation (see Section 4.4.1) when using the system parameters listed in Table 4.1. Additional modifications were incorporated by including a crank-slider transmission mechanism to alter the rest length and a motor model to match power limitations of the physical platform. To account for the effect of the four-bar mechanism, the equation describing the leg-length variation (4.6) was modified to:

\[
\zeta_0 = l_{dev} \cos \theta + \sqrt{l_{cup}^2 - l_{dev}^2 \sin^2(\theta)} + l_{leg},
\]

where \( \zeta_0 \) is the force-free leg length, \( l_{dev} \) is the length of the crank, \( \theta \) is the angle made by the crank measured from the vertical (downwards), \( l_{cup} \) is the coupler length of the four-bar mechanism, and \( l_{leg} \) is the distance from the coupler attachment to the bottom of the foot. This modification was made so the motor could still be operated in a feedforward manner (i.e. rotate at a specific speed) instead of requiring position-based velocity control.

### 4.2.3 Comparison Criterion

To evaluate the effectiveness of the metrics and indicators discussed previously, parameter studies on the variation in energy incorporation \( \omega_{des} \) and in feedback control of the leg angle \( (c) \) will be performed across the three platforms (generalized SLIP, modified SLIP, and experimental). The values for the metrics as a function of these sweeps will be compared in two primary fashions:

1. **Correlation with Mean State Variance**: The Pearson product-moment correlation coefficient will be used as the primary method of assessing the efficacy of each metric/indicator combination. The correlation with mean state variance will give an
Table 4.1: Physical Parameters for Generalized and Modified SLIP Simulations.

<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>$m$</td>
<td>2.11</td>
<td>kg</td>
</tr>
<tr>
<td>$k$</td>
<td>1.92</td>
<td>$kN m^{-1}$</td>
</tr>
<tr>
<td>$b_L$</td>
<td>5.5</td>
<td>$Ns m^{-1}$</td>
</tr>
<tr>
<td>$b_R$</td>
<td>0.5</td>
<td>$Ns rad^{-1}$</td>
</tr>
<tr>
<td>$l_{nom}$</td>
<td>0.298</td>
<td>m</td>
</tr>
<tr>
<td>$l_{dev}$</td>
<td>0.020</td>
<td>m</td>
</tr>
<tr>
<td>$l_{cup}$</td>
<td>0.144</td>
<td>m</td>
</tr>
<tr>
<td>$l_{leg}$</td>
<td>0.154</td>
<td>m</td>
</tr>
</tbody>
</table>

indication of how effectively a given metric can predict the recovery rate of a SLIP-like system across the range of the parameters in question. Mean state variance was chosen to be the primary metric to compare against because it is formulated by assessing how close the system can stay to the limit cycle in the face of continuous, random perturbations. Thus, in order to remain close to the periodic orbit, the system must be able to rapidly recover from perturbations, which would be well characterized by the mean state variance.

2. Location of the Minima in Relation to Mean State Variance: While the correlation to mean state variance can be used to demonstrate the predictive capabilities of a given metric, if that metric is to be used in the design and optimization of a robotic platform, the ability to accurately determine the parameter set with the best disturbance rejection capabilities is essential. Thus, the secondary method of assessing the efficacy of each metric/indicator combination will be to compare the locations of the minima with respect to the mean state variance. This will ensure that the use of a given metric as an objective function will result in the proper parameter set being found.

### 4.3 Methodology and Results

#### 4.3.1 Simulations and Metric Computations

For both the generalized SLIP model and the robot-specific version, numerical simulations were utilized to compute the disturbance rejection metrics of interest and quantify locomotion performance over rough terrain. Simulations were developed and performed utilizing the Runge-Kutta integrator, *ode45*, available in MATLAB. The event functionality
ode45 was utilized to detect instances of leg lift-off and touch-down to a tolerance of $1 \times 10^{-10}$. Model parameters in simulations were set to values characteristic of the robot examined in this study, which are listed in Table 4.1. A Levenberg-Marquardt algorithm, as implemented by the `fsolve` function in MATLAB, was employed to identify periodic orbits, or fixed points, by driving the difference between the initial states and those at the next leg touch-down instant to zero.

For the both the generalized and robot-specific SLIP model, fixed points were found for parameter families specified by separately varying of the $\omega_{des}$ parameter and the $c$ parameter. For $\omega_{des}$, this was accomplished by holding the $c$ and $\beta_{TD}^{des}$ parameters constant while sweeping the range of $\omega_{des}$ for which stable fixed points could be found. To ensure that locomotion performance results were not specific to a particular parameter family, parameter sweeps were performed over three different values of $\beta_{TD}^{des}$ (1.2, 1.3, and 1.4 rad). The same procedure was followed for $c$ as well, with $\omega_{des}$ and $\beta_{TD}^{des}$ being held constant through the sweep. Three different values of $\omega_{des}$ were examined, as with the previous sweep, to ensure that performance characteristic were not specific to the specific parameter family chosen.

The stable periodic gaits identified were subsequently utilized to compute the metrics of interest: mean state variance, maximum eigenvalue magnitude, gait sensitivity norm, two-step decay ratio and settling time. The mean state variance of each fixed point was computed from a simulation run over 150 step disturbances of randomly distributed terrain with each step height selected from a uniform distribution of heights between zero and a maximum height and subject to the constraint that the mean height of the 150 steps must be half the maximum step height. The maximum eigenvalue magnitudes were found by computing the Jacobian of the linearized map, as discussed in Section 4.1.2. The gait sensitivity norm was computed by finding the characteristic matrices and vectors described in Section 4.1.3 and applying (4.3). A two-step decay ratio was found by applying (4.5) with $n = 2$. Two simulation were run, one with a drop step and one with a raised step, and the three strides after the step were used for the calculation. The settling time was found by running the simulation over a drop and raised step, separately, and continuing to run the simulation forward until the desired indicator converged within 1% of the steady-state value. For
the metrics requiring gait indicators, calculations were made separately with the following indicators: leg angle at lift-off ($\beta_{LO}$), stance duration ($t_{stance}$), system energy at touch-down ($E_{TD}$), and the distance of the apex state from the limit cycle ($||\Delta X^{AP}||_2$). To account for variability in the metrics due to the size of the perturbation, all metrics were computed for step sizes (or a maximum step) of 0.02, 0.04, 0.06, and 0.08 m.

### 4.3.2 Simulation Results

Figs. 4.2 and 4.3 show the computed values for several of the metric/indicator pairs as a function of the control parameters $\omega_{des}$ and $c$, respectively. The metrics shown in these figures, though not necessarily the ‘best’ measures of disturbance rejection, showed a greater variation in their characteristic behavior with respect to the parameter being swept than other combinations of metrics and indicators. This is a necessary condition since invariance of the metric would indicate that it is not capturing the changes in behavior that accompany the parameter variation.

The progression of the mean state variance as a function of $\omega_{des}$ and $c$ for both the generalized SLIP model and the robot-based SLIP model were compared with the change in each stability metric/indicator pair through computation of correlation coefficients. For each step size investigated, the Pearson product-moment correlation coefficient was computed to determine the correlation between the changes in the mean state variance and

![Figure 4.2: Disturbance rejection metrics as a function of $\omega_{des}$ as found by the modified SLIP simulation. The horizontal axis for each of the plots is $\omega_{des}$, while the vertical axis is the value of the metric calculated for the respective plot.](image-url)
Each metric along the gait family. The resulting correlation coefficient values were squared and averaged over all parameter sweeps and maximum step heights to yield the values in Table 4.2. Consideration of these results serves as an initial indication of how well the various metric/indicator pairs can approximate the mean state variance. However, since it is crucial to not only measure disturbance rejection in simulation, but on experimental systems as well, it is necessary to consider a similar study on a physical platform.

Figure 4.3: Disturbance rejection metrics as a function of $c$ as found by the modified SLIP simulation. The horizontal axis for each of the plots is $c$, while the vertical axis is the value of the metric calculated for the respective plot.
Table 4.2: Correlation Between Metric(Indicators) and Mean State Variance for SLIP Simulations

<table>
<thead>
<tr>
<th>Metric (Indicator)</th>
<th>Generalized SLIP ($\omega_{des}$)</th>
<th>Generalized SLIP ($c$)</th>
<th>Modified SLIP ($\omega_{des}$)</th>
<th>Modified SLIP ($c$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Maximum Eigenvalue</td>
<td>0.761</td>
<td>0.733</td>
<td>0.334</td>
<td>0.593</td>
</tr>
<tr>
<td>GSN ($\beta^{LO}$)</td>
<td>0.735</td>
<td>0.879</td>
<td>0.909</td>
<td>0.593</td>
</tr>
<tr>
<td>GSN ($T_{Stance}$)</td>
<td>0.900</td>
<td>0.103</td>
<td>0.899</td>
<td>0.403</td>
</tr>
<tr>
<td>GSN ($E_{TD}$)</td>
<td>0.580</td>
<td>0.549</td>
<td>0.521</td>
<td>0.050</td>
</tr>
<tr>
<td>GSN ($</td>
<td></td>
<td>\Delta X^{AP}</td>
<td></td>
<td>^2$)</td>
</tr>
<tr>
<td>$D_{R2}$ ($\beta^{LO}$)</td>
<td>0.238</td>
<td>0.271</td>
<td>0.466</td>
<td>0.323</td>
</tr>
<tr>
<td>$D_{R2}$ ($T_{Stance}$)</td>
<td>0.562</td>
<td>0.047</td>
<td>0.609</td>
<td>0.278</td>
</tr>
<tr>
<td>$D_{R2}$ ($E_{TD}$)</td>
<td>0.880</td>
<td>0.005</td>
<td>0.820</td>
<td>0.119</td>
</tr>
<tr>
<td>$D_{R2}$ ($</td>
<td></td>
<td>\Delta X^{AP}</td>
<td></td>
<td>^2$)</td>
</tr>
<tr>
<td>TS ($\beta^{LO}$)</td>
<td>0.268</td>
<td>0.429</td>
<td>0.019</td>
<td>0.714</td>
</tr>
<tr>
<td>TS ($T_{Stance}$)</td>
<td>0.242</td>
<td>0.074</td>
<td>0.013</td>
<td>0.025</td>
</tr>
<tr>
<td>TS ($E_{TD}$)</td>
<td>0.447</td>
<td>0.585</td>
<td>0.604</td>
<td>0.178</td>
</tr>
<tr>
<td>TS ($</td>
<td></td>
<td>\Delta X^{AP}</td>
<td></td>
<td>^2$)</td>
</tr>
</tbody>
</table>
Table 4.3: Dynamically Scaled Parameters for the Hopping Robot

<table>
<thead>
<tr>
<th>Property</th>
<th>Human</th>
<th>30% Scale</th>
<th>Robot</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mass</td>
<td>80.0kg</td>
<td>2.16kg</td>
<td>2.11kg</td>
</tr>
<tr>
<td>Leg Stiffness</td>
<td>20.0kN m⁻¹</td>
<td>1.80kN m⁻¹</td>
<td>1.92kN m⁻¹</td>
</tr>
<tr>
<td>Leg Length</td>
<td>1.00m</td>
<td>0.300m</td>
<td>0.298m</td>
</tr>
</tbody>
</table>

4.4 Experimental Procedure and Results

4.4.1 Experimental Platform

In order to evaluate the effectiveness of the proposed disturbance rejection on a physical system, a single-legged hopping robot capable of sagittal plane locomotion was utilized [80]. The physical design, shown in Fig. 4.4, was chosen to provide a reasonable resemblance to the SLIP model, utilizing a simple mechanical design. The leg parameters specified via dynamic scaling to match a 30% scale human runner, by length, making the size more manageable than a human-sized platform, as shown in Table 4.3. The robot was driven by a pair of DC brushed motors. One was employed to drive the crank-slider mechanism, extending and contracting the nominal leg length in a sinusoidal fashion; the other was utilized to actuate the hip angle, driving the leg to the desired touch-down angle during flight. The body was designed such that the center of mass coincided with the hip joint, and the moment of inertia was kept minimal to preserve the approximation of the body as a point mass.

The robot was designed to run around a circular track while attached to a boom that rotates about a center pivot. This setup restricts the robot to a spherical workspace around the center pivot. A wooden platform was used for the track with a rough, carpeted surface covering the wood for traction. Steps were made out of wooden risers and used the same carpet as the running surface.

Sensory data was acquired via four quadrature encoders and a contact switch. Both of the motors were equipped with an encoder which were used in real-time to track the motor positions to an accuracy of $2 \times 10^{-4}$ rad. Additional encoders were mounted to the platform which recorded the angular position along the track and angular height above the track of the robot during operation to a resolution of $2 \times 10^{-4}$ rad ($\sim 6 \times 10^{-4}$ m) and $4 \times 10^{-4}$ rad ($\sim 1.2 \times 10^{-3}$ m when the boom is level), respectively, though the track and boom data was
used solely for analysis purposes. The final sensor, a SPST contact switch, was mounted to the bottom of the foot and used to determine the phase (i.e. flight, stance) and sense the occurrence of touch-down and lift-off events.

For additional information on the mechanical and electrical design of the robot and platform, see [80].

4.4.2 Experimental Procedure

In a manner similar to that employed for the simulations, the physical robot was tested when running on both step disturbances and variable height terrain. The robot was subjected to three sets of trials. In each trial set, the ordering of the control parameters was
randomized to block, or compensate, for potential mechanical wear or drift in sensory data during operation.

In a similar fashion to the SLIP simulations, the parameter families characterized by sweeping $\omega_{des}$ while holding $c$ and $\beta_{TD}^{des}$ constant and sweeping $c$ while holding $\omega_{des}$ and $\beta_{TD}^{des}$ constant were examined. For the sweep across $\omega_{des}$, the parameter was varied between 5.0 rad s$^{-1}$ to 7.5 rad s$^{-1}$ in increments of 0.5 rad s$^{-1}$. For the sweep across $c$, the parameter was varied between 0.2 and 0.7 in increments of 0.05 with two additional values 0.025 above and below the extrema, respectively, to allow for a higher resolution in the range at which the stability is expected to change rapidly.

Each trial set consisted of a set of baseline runs, a set of random terrain trials, and a set of step trials. To capture the baseline, each of the parameter sets were run for approximately 80 steps (about 30 m or 4 loops around the track) three separate times. The random terrain was run three times for each parameter set for about 40 steps (2 loops). Lastly, the robot was run over 8 cm up/down steps. Each parameter value was run 3 times over the steps, each time traversing at least two up and two down steps.

For the baseline, the robot ran on smooth carpeted terrain. For the random terrain the track was divided into 16 (0.5 m long) segments. Each segment’s height could be altered in 2 cm increments resulting in terrains ranging from 0 to 8 cm, with a mean track height of 4 cm. Three segments of the track were built for each height, and the segments were ordered randomly. The same random sequence of step height was used for all trials. For the step perturbations, half of the track was raised to 8 cm and the remainder of the track was set at 0 cm.

Fig. 4.5 shows a sample from one step trial in which the robot, running at steady-state, encounters a step down that perturbs the system and then converges back to the periodic gait. While this example shows the behavior of the COM trajectory following the disturbance, other state information, such as velocity, heading angle, and leg angle, behaved in a qualitatively similar fashion. From the series of trials run over the baseline, random, and step terrains, the stability metrics were calculated.

The gait sensitivity norm, decay ratio, and settling times for the gait were recorded for each indicator ($\beta^{LO}, T_{stance}, Energy_{TD}, ||State_{apex}||_2$) for each of the different parameter
values tested. The gait sensitivity norm, decay ratio, and settling time were calculated from the step perturbation, and the mean state variance was computed from the random terrain trials.

Mean state variance, decay ratio and settling time were calculated using the same formula applied in simulation. Gait sensitivity norm was calculated using the method presented in [69] for the discrete response of the gait indicator $g$ to a single disturbance $e_o$ by:

$$\left\| \frac{\partial g}{\partial e} \right\|_2 = \frac{1}{|e_0|} \sqrt{\sum_{i=1}^{q} \sum_{k=0}^{\infty} (g_k(i) - g_\infty(i))^2},$$  \hspace{1cm} (4.9)$$

in which $g_k(i)$ is the value of the $i$th gait indicator $k$ steps after the disturbance has occurred and $q$ is the number of gait indicators. In theory, the number of steps, $k$, should be taken to infinity, but in practice we only considered the first eight steps after the perturbation.
4.4.3 Experimental Results

The experimental values of several metrics as a function of $c$ are shown in Fig. 4.6. The metrics shown allow for easy comparison with the simulation data given in Figs. 4.2 and 4.3. Maximum eigenvalue magnitude was left out of the plot since it was not found on the experimental platform. The decay ratio calculation with $\beta^{LO}$ as an indicator was also left out because the behavior showed a qualitatively similar median to the $||\Delta X^A||$ indicator, but with greater error bars. The comparison of the efficacy of the metrics was done in a similar manner to the simulations, with Pearson product-moment correlation coefficient being calculated between the mean state variance and each metric within the parameter family. The resulting correlation coefficient values were squared and averaged to yield the values in Table 4.4.

A second examination of the correspondence of the various metrics to the mean state variance investigated the location of the minima for each metric during the parameter sweep of $c$. For each metric, the minimum mean value was determined, following which t-tests were performed that calculated the likelihood of a lower minimum mean occurring at each tested value of the control parameter using an $\alpha$ of 0.05. These t-tests established which control...
Table 4.4: Correlation Between Metrics (Indicators) and Mean State Variance for Experimental System

<table>
<thead>
<tr>
<th>Metric (Indicator)</th>
<th>Robot ($\omega_{des}$)</th>
<th>Robot ($c$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>GSN ($\beta^{LO}$):</td>
<td>0.93</td>
<td>0.90</td>
</tr>
<tr>
<td>GSN ($T_{Stance}$):</td>
<td>0.64</td>
<td>0.54</td>
</tr>
<tr>
<td>GSN ($E^{TD}$):</td>
<td>0.57</td>
<td>0.81</td>
</tr>
<tr>
<td>GSN ($</td>
<td></td>
<td>\Delta X_{AP}</td>
</tr>
<tr>
<td>$DR_2$ ($\beta^{LO}$):</td>
<td>0.11</td>
<td>0.80</td>
</tr>
<tr>
<td>$DR_2$ ($T_{Stance}$):</td>
<td>0.11</td>
<td>0.73</td>
</tr>
<tr>
<td>$DR_2$ ($E^{TD}$):</td>
<td>0.96</td>
<td>0.69</td>
</tr>
<tr>
<td>$DR_2$ ($</td>
<td></td>
<td>\Delta X_{AP}</td>
</tr>
<tr>
<td>TS ($\beta^{LO}$):</td>
<td>0.00</td>
<td>0.75</td>
</tr>
<tr>
<td>TS ($T_{Stance}$):</td>
<td>0.69</td>
<td>0.12</td>
</tr>
<tr>
<td>TS ($E^{TD}$):</td>
<td>0.48</td>
<td>0.11</td>
</tr>
<tr>
<td>TS ($</td>
<td></td>
<td>\Delta X_{AP}</td>
</tr>
</tbody>
</table>

Table 4.5: Likely Metric (Indicator) Minima Locations for Sweep of $c$

<table>
<thead>
<tr>
<th>Metric (Indicator)</th>
<th>Likely $c$ Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>MSV:</td>
<td>0.225, 0.250</td>
</tr>
<tr>
<td>GSN ($\beta^{LO}$):</td>
<td>0.250, 0.350</td>
</tr>
<tr>
<td>GSN ($</td>
<td></td>
</tr>
<tr>
<td>DR ($</td>
<td></td>
</tr>
</tbody>
</table>

Parameter values were likely to have a mean at or below the minimum mean found during trials. The potential locations of the minima for the metric/indicator combinations used in Fig. 4.6 are shown in Table 4.5. The settling time was not included since, by inspection, it is clear that the minimum is not located near the location for mean state variance.

4.4.4 Robustness to Noise

During experimentation it was noted that the system did not always converge to a limit-cycle value prior to encountering a step. Motivated by this observation, a simulation analysis of the effect of this source of noise on the various metrics was performed. Perturbations were applied to the apex height of the system in the step prior to undergoing a disturbance. The resulting effect on the metrics were calculated and found to be comparable with the relative levels of noise seen in the experimental data in Fig. 4.6.
The mean state variance showed less than 1% variation for apex height perturbations up to 2 cm and less than 5% up to 5 cm. On the other hand, the 2-step decay ratio exhibited much more sensitivity to noise, showing variations up to 8% when the apex height was perturbed by 2 cm and near 40% for a perturbation of 5 cm. The relatively large variation in decay ratio stems from the inclusion of the indicator for the first step in the denominator of the definition of the decay ratio (4.5). The settling time was on par with decay ratio in the lack of robustness to noise, showing over 15% deviations in the metric when the limit cycle was perturbed by up to 2 cm and 30% for perturbations of 5 cm. This is attributable to the discrete nature of this metric, with small additions of noise occasionally resulting in large jumps in the metric. The gait sensitivity norm does not suffer from the drawbacks exhibited by decay ratio and settling time, and showed variations of less than 0.05% for apex height perturbations up to 5 cm.

4.5 Discussion of Results

An inspection of Tables 4.2 and 4.4 indicates that there is a fair amount of variation in the results between the general SLIP model, the more detailed simulation, and the physical robot when looking at both $\omega_{des}$ and $c$. This is not surprising considering the differences in complexity between the systems. Differences between the general SLIP model and the robot-specific simulations are likely attributable to the altered dynamics resulting from physical damping and a more complex transmission mechanism.

Another source of the differences in the correlation of the metrics is likely due to comparatively large amount of noise in the physical system. The track is not perfectly smooth, flat, or rigid. Occasional foot scuffs and other non-smooth ground contacts introduce additional variation into the data.

4.5.1 Controller Parameter Effects

The peculiarities of the AER control strategy may also influence the relative effectiveness of one metric over another. In particular, earlier studies have shown that the adaptive touch down controller achieves gait restabilization via leg angle variations over the first few steps, which decay quickly. These artifacts are more likely to negatively impact the decay ratio
than the GSN, as the decay ratio only incorporates the first few steps after a perturbation, while the GSN considers up to ten, as shown in the lower variation in the experimental data in Fig. 4.6.

By examining two different AER control parameters, \( \omega_{des} \) and \( c \) we get a better picture of how the various combination of metrics and indicators predict the system behavior. The \( \omega_{des} \) term is related to the resulting speed of the robot running and consequently, the energy of the system, and changing \( \omega_{des} \) results in different fixed points which have different dynamic characteristics. The \( c \) term on the other hand, changes the amount of control effort applied in response to deviations away from the equilibrium conditions, which therefore directly affect the recovery rate. Some metric/indicator combinations are more effective for variations in \( \omega_{des} \) than \( c \), or vice-versa. For example, TS (\( \beta^{LO} \)) and GSN (\( ||\Delta X^{AP}||_2 \)) have a good correlation across all platforms on with respect to variations in the \( c \) term, but are poor for \( \omega_{des} \). On the other hand GSN (\( T_{Stance} \)) works better for \( \omega_{des} \) than \( c \). The performance of other combinations is mixed from one platform to another, but a few are consistently good for both \( \omega_{des} \) and \( c \).

### 4.5.2 \( R^2 \) Correlation

With these caveats in mind, a comparison of the \( R^2 \) correlation coefficients indicates that the gait sensitivity norm with the \( \beta^{LO} \) and \( T_{Stance} \) indicators and the two-step decay ratio with the \( E^{TD} \) and \( ||\Delta X^{AP}||_2 \) indicators are the metrics with the highest mean \( R^2 \) value across all six test cases. Of these, the GSN (\( \beta^{LO} \)) and DR\(_2\) (\( ||\Delta X^{AP}||_2 \)) are the only two that have correlations > 0.80 in at least half of the cases.

For dynamically different gaits due to varying \( \omega_{des} \), leg indicators perform better for the gait sensitivity norm across all platforms while body indicators typically perform better for the decay ratio and settling time. It is worth noting that \( P \) in the discrete Lyapunov matrix equation is the observability grammian (for the GSN calculation). \( P \) is positive definite, implying observability, indicating that input/output relationship in the gait sensitivity norm implicitly contains a more information via these leg indicators than might be the case in the other metrics.
4.5.3 Minima Location

Across all three platforms, the mean state variance minimum changes from the left side of the parameter range swept for $\omega_{des}$ to the right side for $c$. For both of these cases, the gait sensitivity norm, settling time, and decay ratio with $\beta^{LO}$ have shapes similar to the mean state variance, and have their minimum (best) value near the corresponding parameter value for the mean state variance. As shown in Section 4.4.3, experimentally found minima locations are in agreement with the simulation results. Both GSN ($\beta^{LO}$) and GSN ($||\Delta X^{AP}||_2$) show a strong correspondence in expected minima location with the mean state variance, demonstrated by the significant overlap of the expected minima location, which can be seen in Table 4.5. (The combinations not shown in Figs. 4.2, 4.3, and 4.6 have lower $R^2$ values, qualitatively different shapes and/or have minima that do not correlate to the mean state variance as well as the examples shown.) Although the minima is on the ‘wrong’ side for DR$_2$ ($||\Delta X^{AP}||_2$), the shape of the curve (especially in Fig. 4.2) seems to match the eigenvalues better than the mean state variance. This may make sense as two-step decay ratio has some correspondence to eigenvalues with large perturbations.

Although DR$_2$ ($\beta^{LO}$)’s correlation factor is lower than some of the other metric/indicator combinations shown, the sharp delineation and close proximity of the minimum to that of the mean state variance is compelling. This type of curve is likely to better serve the designer performing an optimization or a parameter variation study in an effort to improve the disturbance rejection of the system. For example, if trying to maintain a specific dynamic gait (for a constant $\omega_{des}$ and therefore speed), this metric could be utilized to determine the best $c$ value that would result in the fastest recovery rate (and therefore the minimum expected variance in running over rough terrain). Alternatively, for a constant $c$ value, one could select the optimal gait, which will be different dynamically, by searching for the $\omega_{des}$ value that minimizes a particular gait indicator. As average terrain heights change, this could essentially provide a gain scheduling mechanism to either change speeds or control effort to maximize locomotion performance (via recovery rate). The correlations in the minima of these indicators with those of the mean state variance in locomotion over rough...
terrain may prove helpful in future efforts that attempt to balance disturbance rejection with energetic efficiency.

4.6 Conclusions

This chapter describes the first rigorous comparison of multiple disturbance-rejection metrics and indicators for SLIP-like running on both simple and complex models and as well as on a physical platform. Commonly used stability metrics (e.g. maximum eigenvalues and settling time) as well as more recently proposed approaches (e.g. gait sensitivity norm) and novel metrics (e.g. two-step decay ratio) were recorded for parameter sweeps performed on the representative systems. For the metrics for which it is relevant, the effect of using various gait indicators on these metrics was also considered. In addition, the mean state variance was proposed and utilized as a benchmark metric to evaluate how well these other metrics, which are based on the response to a single perturbation, correspond to the system’s behavior over a continuous series of perturbations akin to that of running over rough terrain.

A comparison of TS, DR, and GSN indicates that, for the AER control system utilized in the simulations and on the robot, distinct combinations of metrics and indicators produce a range of predicted stability behaviors. These results indicate that leg-state indicators ($\beta^{LO}$ and $T^{stance}$) with the GSN and body-state indicators ($E^{TD}$ and $||\Delta X^{AP}||_2$) with the two-step DR demonstrate good correlation to the benchmark MSV values for all three systems studied. The correlation across both simulation and experimental systems suggests that either of these methods of measuring stability may be applicable for SLIP-like running robots in general.
CHAPTER 5

DEVELOPMENT OF A MULTI-MODAL LEGGED PLATFORM

The following sections address the progress that has been made towards the development of a dynamical, multi-modal legged platform capable of exhibiting running and climbing behaviors similar to the biologically-inspired templates for these modalities. Section 5.1 begins by motivating the synergy between the underlying models and presents a 2D model that was used to guide the development process and assists in control and parameter studies. Section 5.2 follows by introducing the SCARAB (Scansorial and Cursorial Ambulation with a Robust, Adaptive roBot) platform and detailing of the mechanical and electrical design.

Sections 5.3 and 5.4 present the preliminary climbing studies aimed to characterize SCARAB’s behavior on a vertical substrate. This is followed in Section 5.5 by the discussion of a series of modifications made to improve the ability of the platform to exhibit running behaviors, in addition to the climbing behaviors previously analyzed. After incorporating these alterations, Sections 5.6 and 5.7 detail the a study on SCARAB’s behavior on level ground. Following the results of this section, further modifications are made to the platform, following which the running and climbing behaviors of the updated platform are characterized in Section 5.8. Section 5.9 concludes the chapter with a summary of the results and a discussion of contributions afforded by SCARAB.

5.1 Modeling and Dynamic Simulation

The motivation behind SCARAB was to field a rapid and robust platform able to traverse level, vertical, and included surfaces. To guide the development of this platform, two
Table 5.1: Parameter Values for Scaling the Reduced-Order Models to 2kg

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Model Values†</th>
<th>Scaled Value‡</th>
<th>Robot Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mass (kg)</td>
<td>0.002 / 0.0025</td>
<td>2 / 2</td>
<td>1.88</td>
</tr>
<tr>
<td>Leg Stiffness (Nm$^{-1}$)</td>
<td>6 / 3.5</td>
<td>600 / 300</td>
<td>640 / 320</td>
</tr>
<tr>
<td>Leg Length (m)</td>
<td>0.0154 / 0.017</td>
<td>0.154 / 0.158</td>
<td>0.200</td>
</tr>
<tr>
<td>Actuation Length (m)</td>
<td>0.0092 / –‡</td>
<td>0.092 / –‡</td>
<td>0.088</td>
</tr>
<tr>
<td>Stride Frequency (Hz)</td>
<td>9 / 10</td>
<td>2.85 / 3.28</td>
<td>Varies</td>
</tr>
<tr>
<td>Expected Velocity (ms$^{-1}$)</td>
<td>0.20 / 0.25</td>
<td>0.63 / 0.76</td>
<td>Varies</td>
</tr>
</tbody>
</table>

† For the model and scaled values, the left value corresponds to the FG (climbing) model value while the right value corresponds to the LLS (running) model value.
‡ There is no specified actuation length for the LLS model due to it being energetically conservative.

Biologically-inspired models were used as templates: the Full-Goldman climbing model (see Section 2.1.3) and the Lateral Leg Spring running model (see Section 2.1.2). The similarities between these two models bely a natural synergy, as several animals have demonstrated behaviors characteristic to both models depending on surface inclination [11,106]. Since both of these models were formulated for a 2g system, it was necessary to apply dynamic scaling to determine the appropriate system parameters a platform at the target size of 2kg, the results of which are shown in Table 5.1. While the parameter values for the scaled models were similar in most aspects, the desired leg stiffnesses show significant deviation between running and climbing, necessitating the consideration of a variable stiffness mechanism for the physical platform.

After determining the dynamically scaled parameter values for the target platform, a dynamic simulation was developed in Working Model 2D® [107]. The simulation has a quadrupedal configuration and was designed with mass distributed in the legs and actuation mechanisms to better model the physical system. This simulation served as a tool for motor selection, controller design, and investigation of the effect of varying rear leg posture. Furthermore, consideration of the simulation dynamics allowed for further verification of the dynamic similarity relations discussed in Chapter 3. It will additionally be used in performing parameter variation studies and examining transitions between climbing and running.
5.2 Physical Platform Development

After demonstrating that the dynamic simulation preserved the climbing behavior of the FG model, the first prototype of SCARAB was designed, consisting of four legs for actuation and attachment, an electronics package for control of the robot, and a central body to connect the individual components. The fully assembled platform, shown in Fig. 5.1A, is approximately 50cm long and 30cm wide, depending on the configuration of the legs, and has a mass of 1.88kg.

Each leg, shown in Fig. 5.1B, is actuated via a Maxon RE-max 24 motor (Maxon
222049) with a 24:1 planetary gearhead (Maxon 14397). The motor is in series with a 3:2 bevel gear set, which drives a crank-slider mechanism to vary the rest length of the leg. This mechanism is used to add (or remove) energy during locomotion. Additionally, each leg has a linear spring in series with the crank-slider, which reduces the peak ground reaction forces, lowering the stress on the leg and reducing peak loads on the motor. It also assists with attachment, allowing the loading on the foot to increase gradually. As mentioned in Section 5.1, the shin, which houses the leg spring, was designed so it exhibited a different effective stiffness during running and climbing in a manner similar to that used on RiSE v1 [108]. The mechanism utilized for this was a slider braced between two sets of compression springs, which can be seen in Fig. 5.1B. Since the springs were not attached to the slider, moving the slider one way or the other would only engage one set of springs. By appropriate selection of the spring stiffnesses on both sides, the leg could behave with two separate effective stiffnesses depending on the direction of compression. The design of the leg is intentionally similar to a previous sagittal plane runner [80], facilitating the future implementation of SLIP-like running on level and sloped surfaces.

A foot is attached to the end of each leg for attachment to the climbing substrate. At this stage of development, the goal of the attachment scheme was to utilize a simple, passive mechanism that would enable reliable attachment without impacting the dynamic performance of the robot. To this end, the foot is designed to utilize a hook-and-loop attachment mechanism. Each foot uses a single toe to allow the attachment point to function as a pin joint. The toe is a debarbed fish hook that has been bent to allow the point to catch the climbing surface. While this mechanism is fairly simple, it demonstrates directional adhesion in a manner comparable to that utilized by animals and previous climbing robots [108–110]. Additionally, the length of the toes on the front leg and back leg are different to position the robot at approximately a 10° angle relative to the climbing substrate. This allows the robot to pull itself towards the wall while in stance to improve the success of attachment at touch-down. The climbing substrate is a 2.5m by 1.25m vertical wooden wall with Berber carpet affixed to the climbing surface.

For platform operation, independent control of four actuators at a 1kHz update rate was desired. In addition, the control system needed to be compact and light-weight. This
motivated the development of a custom electronics package around the Gumstix Overo® Fire. This controller was chosen due to its small footprint, high clock frequency, expandable memory capacity, and the availability six pulse width modulation (PWM) and analog-to-digital lines. The primary drawback was the lack of sufficient general purpose input/output (GPIO) lines. To overcome this limitation, a custom expansion board, shown in Fig. 5.1C, was designed based on the architecture of several Intel microprocessors [111], employing a multiplexed address/data bus and a separate control bus. This configuration is able to address up to 16 8-bit devices (or up to 128 individual GPIOs) to communicate with the central processor, though the current design only utilizes 4 of these device addresses. The electronics package enabled the utilization of two dual quadrature decoders and four single channel motor drivers to track and control the motors, while weighing only $298g$ and fitting into a $11cm \times 13cm$ footprint.

The body to which the legs and electronics are attached is a $30cm \times 30cm$ aluminum frame. The legs are each attached at the hips, located $12.5cm$ in both fore-aft and lateral directions from the center of the frame, while the electronics are mounted directly over the center of the body. This configuration places the geometric center of mass of the platform approximately in the center of the robot.

For both climbing and running, SCARAB utilizes a trotting gait, in which the front right and rear left legs are in stance while the front left and rear right legs are in flight and vice versa. To control positioning of the legs and to maintain the phase offset of the trotting pairs, position control of the individual leg motors is utilized. Each motor is given a desired trajectory that is prescribed by the stride frequency without any information about the position of the other motors. The motor tracks this desired trajectory using a proportional controller.

### 5.3 Experimental Procedure for Climbing Characterization

The aim of the experimental tests described in this work is two-fold. The first goal is to verify the similarity of the physical platform, simulation, and the FG climbing model. The second is to examine the effect of rear leg configuration on climbing behavior. The development of the quadrupedal SCARAB enables the examination of fore-aft leg specialization,
which has been shown to be beneficial for running robots but had yet to be tested in the
scansorial regime.

To quantify the behavior of the platform, experimental data was gathered using mo-
tion capture and current and voltage sensing. Whole body position and velocity data was
obtained via a motion capture system using a high-speed digital camera (Casio Exilim EX-
F1) that tracked 2 LED markers located above and below the center of mass of the robot.
Motion tracking data was captured at 300\(fps\) and analyzed in MATLAB using a custom
point-tracking script. Power consumption was calculated from the current draw of the robot
and the input voltage. A Vektrex™ VCS10 current sensor and a Sparkfun® Logomatic V2
was used to measure and log the current draw of the robot, which was synchronized with the
motion capture data and input voltage to determine the total system power consumption
and efficiency during climbing.

The first experimental goal, verifying the similarity between SCARAB, the quadrupedal
simulation, and the FG model, was performed by comparing center of mass trajectories as
well as fore-aft and lateral velocity profiles over the course of a stride. Since the FG model
utilizes a sprawl angle of 10°, this angle was selected for the sprawl angle of the front legs on
the quadrupedal platforms. A rear sprawl angle of 10° was also selected for this comparison.
For the physical platform, the motion capture system was used to gather data from 15 trial
runs, which were compiled to generate an average center of mass trajectory and velocity
profiles over the course of a stride. In each trial, the robot was placed at the bottom of the
wall and climbed to the top. To help minimize transients, only the data from the final stride
was used for the verification. The simulated data was obtained through forward simulation
of the quadrupedal model. The simulation was allowed 15 strides to reach steady state
before data was gathered from a single stride.

In the second set of experiments, the effects of rear leg posture were investigated. Several
rear leg configurations were examined, as shown in Fig. 5.2. The first case was an outward-
sprawled configuration, which showed similarity to the observed leg orientation of sprawled
posture animals during climbing (though not necessarily their force generation). The second
case was a zero-sprawl configuration, which was expected to demonstrate similar rear leg
function to cockroaches [11]. The third case was an inward-sprawled configuration and
was expected to reproduce ground reaction forces observed in the rear legs of geckos when climbing rapidly [106]. Overall, five rear sprawl angles were examined, ranging from $-20^\circ$ to $20^\circ$ in $10^\circ$ increments, with negative angles corresponding to inward-sprawl and positive angles corresponding to outward-sprawl.

Three behavior characteristics were examined for each configuration: fore-aft velocity, lateral velocity, and efficiency. The fore-aft velocity corresponds to the mean fore-aft velocity during the stride. The lateral velocity was calculated as the root-mean-square lateral velocity during the stride. Finally, the efficiency was determined via specific resistance (SR), calculated by $SR = P/mgv$, where $P$ is the average power consumption during the stride, $m$ is the mass of the robot, $g$ is the acceleration due to gravity, and $v$ is the average fore-aft velocity. Note that lower values for SR are more efficient.

5.4 SCARAB Dynamic Climbing Results

In preliminary experiments, the robot was run on vertical and horizontal surfaces and demonstrated mean speeds of up to $0.17 \text{ ms}^{-1}$ when climbing a vertical surface and up to
0.43 m/s when running across level ground. The climbing results are further discussed below while the running will be more fully examined in future work to allow for an improved analysis of the two locomotion modalities.

The velocity profiles over the course of a single stride for the FG model, the dynamic simulation, and SCARAB are shown in Fig. 5.3. Both fore-aft and lateral velocity exhibit similar profiles for all three. However, the fore-aft speed is lower than predicted by dynamic scaling (see Table 5.1). This is the result of running the quadruped at a 1.5 Hz rather than the dynamically scaled frequency of 2.85 Hz, which was due to failure to establish attachment with the front feet at high stride frequencies. While several factors likely contributed, the most significant is probably that the stiffness of the rear legs being less than the dynamically scaled values. Since the rear toes were made long enough to produce a 10° angle between the body and the wall, they also acted as a cantilever beam in series with the leg, which was not accounted for in the development of the platform. This resulted in a lower natural frequency of the leg, such that when attempting to climb at the dynamically scaled frequency, the rear springs would not be able to return the energy stored during
leg extension. It also pivoted the body about the rear toe attachments, pitching the front feet away from the wall. However, the similarity of the qualitative shape of the profiles demonstrates that SCARAB still exhibits the lateral force generation and oscillating velocity profile characteristic of the climbing template. Furthermore, increasing the speed of the dynamic simulation results in the expected mean climbing speed being realized.

Rear leg sprawl variations were also examined, as shown in Fig. 5.4. From these results, several trends can be noted. First, moving from more outward-sprawled postures to more inward-sprawled postures increases vertical climbing speed by 10%. Second, zero-sprawl postures exhibit the lowest peak lateral velocities while a sprawled posture, whether inward or outward, increases the lateral velocity, up to twice the magnitude of the zero-sprawl configuration. While low lateral velocity may at first seem desirable, lateral velocities of approximately half the climbing speed have been shown to correspond to more stable climbing [16]. Third, although the magnitude of the difference is small (only about 1%), a similar trend to lateral velocity is observed for specific resistance. This indicates that climbing with a zero-sprawl posture may slightly improve efficiency while more sprawled postures may be better suited when stability is most essential.

The effects of utilizing an inward-sprawled posture were unable to be tested on the physical robot due to attachment failure. This is a result of out of plane roll that was not captured in the 2D dynamic simulation. The roll results in the front foot missing the climbing substrate when the leg begins contracting, causing the robot to fall off the wall. This effect has been seen in previous dynamic climbing robots and has been dealt with in the past through stabilization via a roll bar extended off the rear of the platform [16]. While the lack of a roll bar keeps the experimental platform from utilizing inward-sprawled configurations, the robot is able to climb successfully with no rear sprawl or outward-sprawled legs, suggesting that the rear legs, when not sprawled inward, reduce out of plane roll.

Results for the outward and zero-sprawl configurations show similar trends to the simulation data. To assess the significance of the trends, two-sample t-tests were performed between the behavior characteristics of the zero-sprawl and the 20° outward-sprawled configurations. A significant decrease in SCARAB’s mean climbing speed \( (p < 0.001) \) was
observed as the sprawl angle was increased from 0° to 20°, as well as increases in both peak lateral velocity ($p < 0.0001$) and specific resistance ($p < 0.001$). It is worth noting that the improvement in efficiency on the physical platform is greater than observed in simulation, and is likely the result of measuring efficiency via electrical power on SCARAB rather than mechanical power, as was done in simulation. These results corroborate the simulation findings and suggest a trade off between efficiency and stability.

### 5.5 Modifications for Level Ground Running

While the original design of SCARAB could successfully climb vertical surfaces at high speeds, its mobility on level ground was limited. To improve its ability to run on all inclines, several design modifications were made, including the adoption of a non-planar body, the addition of compliant hips, and the modification of the attachment mechanism.

The first problem identified with the original version of SCARAB was that the body and legs dragged along the ground when on level or mildly-inclined surfaces. This resulted
Figure 5.5: (A) Non-planar SCARAB body. This modified body incorporates a splay angle $\Psi$, measured between the vertical and transverse axes, to allow the legs to lift off the ground and reset to their nominal position. (B) Compliant hip mechanism that allows the legs to rotate when the leg is in stance and reset during flight. (C) Front foot attachment mechanisms. These are several prototype claws tested for use on the front legs for running. The curved claw was utilized in the final version as it gave sufficient attachment during stance while being able to slide along the running surface to reset without catching during flight. (D) Updated SCARAB platform in its running configuration.
in significant friction and hampered engagement of the claws with the running surface. To elevate the platform, a non-planar body was adopted on which the leg mounts were bent inward to provide splay, as shown in Fig. 5.5A. The splay angle $\Psi$ is defined by the angle between the projection of the leg axis on the transverse plane and the vertical axis. For example, a body with $\Psi = 90^\circ$ would have a planar configuration while a body with $\Psi = 0^\circ$ would be fully upright. This modification results in the claws moving towards or away from the ground during extension and retraction, respectively. This allows each leg to have a flight phase, during which the leg is able to move forward without the claws catching on the running surface. For the subsequently described experiments, a splay angle of $50^\circ$ was utilized, as this provided a sufficient angle to disengage the claws while still keeping the legs at an angle that allowed for significant lateral force generation.

An additional issue with the original design was that the rigid attachment of the hips to the body limited to the front legs to act as pure breaking elements. By modifying the hips to allow rotation, the legs could switch from breaking to accelerating as they rotated during stance. Furthermore, dynamic climbing simulations indicated that the addition of hip compliance would improve vertical locomotion as well. A mechanism was developed that would allow in-plane rotation of the hips, shown in Fig. 5.5B. A pin joint was used at the hip to allow rotation while antagonistic extension springs drove the leg towards the desired touch-down angle. An additional alignment slider was added to restrict out-of-plane motion of the leg.

The final platform modification was the replacement of the front and rear claws and alteration of the running surface. The need to change the attachment mechanism resulted from the front claws engaging in compression during running; however, rotating the front claws caused them to dig in to the running surface and never disengage. To remedy this, bent aluminum plates with a curved front edge, shown in Fig. 5.5C, were utilized that engaged the surface through frictional attachment but could easily be lifted from the surface. A rubberized tile was also used instead of carpet to keep the claws from catching when disengaged. Debarbed fish hooks were still used for the rear claws, though the angle of incidence with the surface was made much steeper to engage the rubber surface.
The resulting platform after modifications is shown in Fig. 5.5D. With the addition of compliant hips, SCARAB still demonstrates effective dynamic climbing, and when using a splay angle of 70° or larger, it is able to climb as well; however, modifications to the claws results in failed attachment on surfaces with inclines greater than 30°. Future platform developments will consider the addition of adjustable splay angles and examine alternative claws to allow SCARAB to utilize both locomotion modalities without manual alterations.

5.6 Experimental Procedure for Running Characterization

5.6.1 Parameter Study

Numerous parameters affect the locomotion performance of SCARAB. These include physical parameters, such as leg stiffness and length, configuration parameters, such as sprawl and splay angle, and control parameters, such as actuation speed and leg phasing. Since the physical parameters of the platform were established via dynamic scaling [16] and their variation has been previously investigated [60], they were omitted from this study. The study parameters were chosen on the basis of the anticipated effect on performance, the ease at which they could be controlled, and their potential for on-the-fly adaptation in future platform iterations. This led to the selection of two parameters to be examined in this analysis: front leg sprawl angle $\beta$ and phase offset angle $\Phi$.

The first parameter, front leg sprawl angle, is the angle between the projection of the front leg axis on the coronal plane and the central body axis, as shown in Fig. 5.6A. The front leg sprawl angle is anticipated to have a significant effect since it directly impacts the direction of the initial braking force in the front legs. Adjustment of the front leg sprawl angle on the current platform is performed by manually adjusting the angle at which the compliant hip mechanism is attached to the body.

The second parameter, phase offset angle, shown in Fig. 5.6B, is the phase difference between the leg extension of the front and rear legs on the same side of the body. Legs on opposite sides of the body were kept 180° out-of-phase for all trials, limiting the examination to symmetric gaits. The phase offset angle is expected to have an impact due to the importance of gait selection at a given actuation speed. While many platforms arbitrarily selected gaits, animals utilize a number of different gaits and often switch depending on
Figure 5.6: (A) Sprawl angle is determined by measuring the angle between the projection of the leg axis on the coronal plane and the central body axis. (B) Phase offset angle is determined by the phase difference in the front and rear extensions on the same side of the body. Legs on the opposite side of the body are mirror by 180°.

their speed [112]. The adjustment of this parameter is straight forward, as the offset can be prescribed directly via the motor controller.

The range of the parameter variations was determined in preliminary studies. For front leg sprawl angles of 90° and greater, the front legs were unable to brake, resulting in the robot tipping over. Additionally, when the front sprawl angle was less than 45°, the front legs could only brake and the claws tended to catch, causing irregular behavior. This prompted the selection of the range of sprawl angles to be from 40° to 80° in 10° increments. For the phase offset angle, initial tests examined a range of offsets from 90° to 270°. For phase offsets of less than 180°, the robot could not maintain a heading and would rapidly veer off-course. This led to three phase offsets to be examined in the experimental parameter study: 180°, 225°, and 270°.

5.6.2 Data Acquisition

To obtain SCARAB performance data, trials were run for each of the 15 configurations (5 sprawl angles and 3 phase angles). Center of mass trajectory data was captured using a two-
camera Vicon Bonita motion tracking system. This system allowed for three dimensional position and orientation data to be recorded at 100Hz. For each configuration of the robotic platform, 10 trials were run, with SCARAB running for approximately 25 strides over the course of each trial. From each trial, 6 consecutive strides were selected as a representative data set for the run. This provided sufficient time for the system to reach steady state while ensuring that the data set is large enough to rule out bias in selecting the strides.

5.6.3 Data Analysis

The data acquired via the Vicon system contained position and orientation data for each stride. To account for differences in the initial heading of the platform, the data was first reoriented such that lateral and vertical displacement over the course of each stride was set to zero. To calculate velocity, a difference formula was used after applying a 3rd order moving average filter using MATLAB’s `filter` function to the position data.

SCARAB’s running performance through the experimental trials was quantified via two methods. First, the mean fore-aft velocity over the course of a stride was compared for each set of sprawl angles and phase offset angles. Second, mean horizontal and fore-aft velocities profiles for the platform were compared to those for the LLS template. Characteristics examined for similarity include the profile shape, the phasing of the fore-aft and lateral velocities, and the relative maxima and variation in the velocities.

5.7 SCARAB Level Ground Running Results

To compare the effects of front leg sprawl angle and phase offset angle on running speed, the mean fore-aft velocity was examined from the experimental data for each parameter configuration and is plotted in Fig. 5.7. From this data, we observe, first, that over the range of front leg sprawl angles examined, the use of a 180° phase offset resulted in reduced fore-aft velocities as compared 225° or a 270° phase offsets with the same sprawl angle. Furthermore, during the 180° phase offset trials, it was observed that the platform exhibited erratic motions, violently shaking the platform, which were not noted for other phase angles. Second, use of a front leg sprawl angle between 40° and 60° resulted in the highest fore-aft velocities, demonstrating mean running speeds around 40cms⁻¹. While the
phase offset angle resulting in the highest mean fore-aft velocity varies with sprawl angle, these differences are not statistically significant.

After considering the speeds of the different platform configurations, the velocity profiles were examined. These profiles were compared to the biologically-inspired LLS template, shown in the first column of Fig. 5.8, to determine the degree to which SCARAB was able to demonstrate similar dynamics to the model. The profiles for the LLS template were adapted from [60], for which the parameter set was chosen to match the horizontal plane dynamics of cockroaches. Five characteristics of the LLS velocity profiles are compared to the robot velocity profiles to ascertain how well the dynamics corresponded.

The first of these characteristics is the number of peaks per stride for the fore-aft and lateral velocities. For the LLS model, two fore-aft velocity peaks and one lateral velocity...
peak occur every stride. The second characteristic is the phasing of the fore-aft and lateral velocities. For the LLS model, the fore-aft velocity peaks are coincident with the maximum positive and negative lateral velocities while the fore-aft velocity minimums correspond to the lateral velocity being zero. The third characteristic is the relative magnitude of the lateral velocity, which is measured as \( v_{\text{lat peak}}/v_{f a \text{ mean}} \), where \( v_{\text{lat peak}} \) is the maximum magnitude of the lateral velocity and \( v_{f a \text{ mean}} \) is the mean fore-aft velocity. For the LLS model, this ratio is 10%. The fourth characteristic is the percent of variation in the fore-aft velocity, calculated as \( (v_{f a \text{ max}} - v_{f a \text{ min}})/v_{f a \text{ mean}} \), where \( v_{f a \text{ max}} \) and \( v_{f a \text{ min}} \) are the maximum and minimum fore-aft velocities, respectively. For the LLS model, this value is 2.5%. The final characteristic considered is the ratio of maximum positive and negative lateral velocities. For the LLS model, they are equal, resulting in a ratio of 1.

The SCARAB velocity profiles were examined for all sprawl angle and phase offset angle combinations; however, as the velocity profile behavior was fairly invariant to sprawl angle, we only present the results for \( \beta = 60^\circ \) here. This sprawl angle was selected for detailed analysis because it was at the midpoint of the examined front sprawl angles and showed some of the fastest speeds. The SCARAB profiles with this sprawl angle and phase offsets...
Table 5.2: Characteristics of the Velocity Profiles for SCARAB Running

<table>
<thead>
<tr>
<th>Profile Characteristic</th>
<th>LLS</th>
<th>Φ = 180°</th>
<th>Φ = 225°</th>
<th>Φ = 270°</th>
</tr>
</thead>
<tbody>
<tr>
<td>Peaks per Stride†</td>
<td>2/1</td>
<td>1/1</td>
<td>2/1</td>
<td>2/1</td>
</tr>
<tr>
<td>Velocity Phasing</td>
<td>0°</td>
<td>–</td>
<td>30°</td>
<td>10°</td>
</tr>
<tr>
<td>Relative Lat Velocity</td>
<td>10%</td>
<td>125%</td>
<td>116%</td>
<td>98%</td>
</tr>
<tr>
<td>F-A Velocity Variation</td>
<td>2.5%</td>
<td>63%</td>
<td>38%</td>
<td>22%</td>
</tr>
<tr>
<td>Lat Velocity Ratio</td>
<td>1</td>
<td>1.83</td>
<td>1.15</td>
<td>1.02</td>
</tr>
</tbody>
</table>

† Fore-aft and lateral peaks per stride are denoted on the left and right side of the slash, respectively.

of Φ = 180°, Φ = 225° and Φ = 270° are shown in the final three columns of Fig. 5.8 and the characteristics of these profiles are reported in Table 5.2.

Both qualitative examination of the velocity profiles and consideration of the profile characteristics show that SCARAB’s dynamics are quite distinct to the LLS model when using a phase offset angle of 180° or 225°. The former is dissimilar in almost every characteristic, with the only similarity being the presence of a single lateral peak per stride. The latter shows the same number of peaks and has a relatively symmetric gait, but does not match the relative magnitudes or phasing well.

Conversely, the velocity profiles and characteristics for Φ = 270° are quite similar to the LLS model. The number of peaks, velocity phasing, and lateral velocity ratio agrees with the qualitative observation that with this phase offset angle, the robot velocity profiles have a similar shape to the LLS model. The relative lateral velocity and variation in fore-aft velocity are still significantly different, though both are much closer to the model’s characteristics than with the other phase offset angles.

With only this coarse parameter sweep, we can conclude that with β = 60° and Φ = 270°, SCARAB is able to run at its fastest speeds with similar dynamics to the LLS model. The two most significant discrepancies between the robot and the model are the large values for relative magnitude of lateral velocity and variation in fore-aft velocity on the platform. The increased variation in fore-aft velocity is in part a result of not fully optimizing the robot parameters. The presence of larger than expected lateral velocities may also have contributed to the high degree of variation in the fore-aft velocity.
It is notable that the relative lateral velocity magnitude is consistently large across all phase offset angles. The platform’s splay angle is likely the cause of this discrepancy. A splay angle of 50° was selected to emphasize the horizontal plane motion, but the experimental results indicate that such a large splay angle may be undesirable. This suggests that a more upright posture for SCARAB will improve the similarity to the LLS model.

5.8 Modified SCARAB Performance

5.8.1 Running

Given the analysis of the previous results, which indicated that a more upright posture may improve the correlation of SCARAB’s motions to the LLS template, system modifications were incorporated to provide SCARAB with a splay angle of 20°. Preliminary experiments with SCARAB running in an upright posture indicated that gait tuning would
be necessary in order to produce consistent running behaviors. Since the original model (see Section 5.1) only captured planar motion, it was deemed unsuitable for tuning the upright behavior since considerable vertical motions were present. This led to the development of a 3D model in Adams\textsuperscript{TM} [113]. In addition to tuning the leg phasing and angles, this model was also used for tuning of the hip stiffness. Using this model, a suitable set of the aforementioned parameters were determined to characterize the performance of SCARAB experimentally.

After tuning the gait in the 3D model, experiments were performed on the physical platform with a splay angle of 20\degree, shown in Fig. 5.9A, using the tuned set of parameters. Data acquisition for the experiments with upright running was recorded using a 10-camera Vicon motion capture system, which captured the center of mass trajectory at approximately 350\textit{fps}. The analysis of the trajectory data was performed in the same manner as previously described for the splayed running experiments. The experimental velocity profiles in the fore-aft, lateral and vertical directions are shown in Fig. 5.10. The vertical motions were included since they further enable the comparison of the system to SLIP dynamics, which had previously been neglected since in a splayed configuration, very little vertical motion was exhibited.

The first observation, and most obvious from a qualitative assessment, is that the clean double peak in the fore-aft velocity for each stride is not as apparent in the upright behavior. While this may appear to be a step back from correlating favorably with the LLS model (compared to the 70\degree splayed configuration), it is not surprising due to the magnitude of the error in the fore-aft data. Furthermore, it is evident that two peaks are present, though they are not of the same magnitude and are slightly shifted. This, in addition to the large standard deviations, is likely due to the limited running space available that precluded a sustained period of steady-state locomotion from which data could be collected.

However, even with these limitations, there are indications that while in an upright configuration, SCARAB is able to not only exhibit better correlation to the LLS model but produce improved performance in general. This is most clearly exhibited by the 48\% increase in fore-aft velocity from a maximum of 45.4 \pm 5.3\textit{cms}^{-1} when using a splay angle of 70\degree to 67.0 \pm 8.0\textit{cms}^{-1} when using a splay angle of 20\degree. Additionally, when considering
the relative lateral velocity, which was the primary motivator for switch to a more upright posture, magnitude decreased from $\sim 100\%$ to $\sim 30\%$. These results indicate that using an upright posture with the SCARAB platform produces a stronger correlation to the underlying biological models.

In addition, several physical limitations may be hindering the performance of the platform and hampering it from reaching its peak performance. The first of these is the spring mechanism. Though it provides the proper stiffness, on the front legs in particular, the springs tend to bottom out, particularly in transitory periods. This impedes SCARAB from reaching steady-state since the impulse of the spring maximally compressing disrupts the system dynamics. An additional limitation is evidenced through ground attachment. While this issue was addressed by using a higher coefficient of friction for the rear feet than the front, there is still a considerable amount of slippage with the rear legs, resulting in energetic losses during the power stroke. By addressing these issues and applying more rigorous optimization of the gait parameters and hip stiffness, SCARAB’s performance on level surfaces should continue to improve to the point that its fore-aft speeds on par with the scaled LLS template (see Table 5.1). Furthermore, these results should indicate a high correlation to the sagittal plane dynamics of the SLIP template.
Figure 5.11: Center of mass trajectory and velocity profiles for SCARAB climbing after incorporating modifications for running. (A) shows the center of mass trajectory in the lateral and vertical directions during climbing. (B) and (C) show the velocity profiles in the vertical and lateral directions, respectively, with the mean shown by the centerline and the error bars showing one standard deviation in the data.

5.8.2 Climbing

Though SCARAB was demonstrated to run effectively after the modifications described above, it was not clear how these changes would affect the climbing behavior. To consider this more closely, SCARAB’s climbing behavior was reexamined with many of the modifications for running incorporated, including a 70° splayed body and compliant hips, as shown in Fig. 5.9B. However, some alterations were still required. First, an alternative set of claws were used to attach to the climbing substrate. Second, the sprawl angle of the legs needed to be adjusted to be more conducive to climbing. Finally, while a compliant hip was used on the front legs, the rear legs required the compliance to be removed. Though all of these changes currently required manual adjustment, mechanisms for autonomous modifications, both passive and active, are under consideration.

To examine the effect of the platform modifications on SCARAB’s climbing performance,
an experimental characterization of the climbing behavior was performed. The platform was run for 10 trials using same trotting gait and a stride frequency of $1.5\,Hz$ were used as in the previous characterization (see Section 5.4). For each trial, center of mass trajectory data was captured and analyze with the 10-camera Vicon motion capture system previously described. The trajectory and velocity profile data from these experiments is shown in Fig. 5.11.

Comparing these results to the preliminary climbing characterization in Fig. 5.3 shows a strong similarity between the original and modified platform. Of first note is that the climbing speed is almost identical, with the original platform showing a mean climbing speed of $16.4\,ms^{-1}$ and the modified version climbing at $16.3\,ms^{-1}$. Furthermore, the overall shape and phasing of the velocity profiles is markedly similar, indicating a that with the system modifications, SCARAB is still climbing in a similar manner as before.

While the performance is on par with the previous characterization of the platform, it is still not achieving the speeds expected by dynamic similarity, and future modifications may bring the platform closer to the predicted performance. It is worth noting that no formal optimization has yet been performed and the current performance has been achieved with limited tuning. A more rigorous consideration of the parameters not intrinsic to the FG model, such as leg phasing and angles\textsuperscript{1}, would likely increase climbing speed and reliability.

\section*{5.9 Conclusions}

This chapter has addressed the design and characterization of SCARAB, the first dynamical robot to exhibit biologically-inspired, legged locomotion on both level and vertical surfaces. It began with the development of 2D dynamical models of the running and climbing behaviors, which were used to demonstrate the dynamic similarity of the proposed platform to the underlying biologically-inspired templates at the robot scale. Additionally, a 3D model was later developed that corroborated these findings and provided a means for considering the unconstrained, whole-body motions.

\textsuperscript{1}While the leg angle adjusted is technically the sprawl angle, since the body is upright, it actually has a minimal effect on the actual sprawl angle. Thus, this parameter has more of an effect on the distance of the center of mass from the wall and the vector along which the front and rear legs pull and push, respectively.
Using these models to guide the design process, the dynamical, multi-modal robot was developed. The performance of this platform was then characterized on both vertical and level surfaces and compared to the design models and underlying templates. Furthermore, as it was a quadrupedal system, SCARAB allowed for the consideration of several research questions related to dynamic locomotion of quadrupedal platforms. First, on vertical surfaces, the effect of leg differentiation was examined, in particular considering how rear leg sprawl influenced the speed and efficiency of climbing. The results from this investigation demonstrated that rather than keeping rear legs sprawled outward, turning them in towards the body centerline and pushing produced higher speed and more efficient climbing behaviors.

Second, on level surfaces, the desired touch-down angle of the front legs and the phase offset between the front and rear legs was investigated. It was found that using a smaller touch-down angle, which also corresponds to increased braking, the running speed actually increased. While counterintuitive, since braking would seem to slow the platform, smaller sprawl angles tended to stabilize locomotion and minimize the pitching that was observed to accompany the platform when using larger sprawl angles. In addition, examination of the effect of phase offset angle demonstrated that by tuning the phase such that the front legs led the rear legs by $\sim 90^\circ$, rather than $180^\circ$ as in a classical ‘trot’, both locomotion speed and correlation to the LLS model could be improved.

In the course of considering the running performance, it was noted that the highly splayed configuration used for running resulted in significantly larger lateral motions than exhibited by the LLS model when selecting a biologically-relevant set of parameters. This motivated the modification of SCARAB to use a more upright posture, which, in addition to showing a better ratio of lateral to fore-aft velocities, was found to produce running speeds 50% faster. Using this modified configuration, the climbing behavior was also investigated and found to be markedly similar to the original version as well as being more amendable to autonomous transition for future work.
CHAPTER 6

CONCLUSIONS

6.1 Dissertation Summary and Conclusions

The work presented in this dissertation has considered several of the contributing factors for the development of mobile, legged platforms capable of high-speed locomotion in multiple regimes. Although previous investigations of biologically-inspired locomotion have yielded a number of highly capable platforms, these systems have been unable to demonstrate the combination of versatility, speed and robustness that is observed in nature. This work attempts to bring our understanding of the factors contributing to dynamical, multi-modal locomotion to a point such that a platform capable of instantiating biologically-inspired, legged locomotion on both level and vertical surfaces is possible.

We began by considering how scale affects locomotion performance and, in particular, how similar locomotion behaviors can be preserved between models and platforms of different sizes. Using the concept of dynamic similarity, a set of scaling relationships was developed that specifies a set of system parameters that will preserve the locomotion characteristics at any arbitrary scale. While certain constraints must be followed in this process, the end result provides flexibility in both design and implementation when attempting to scale dynamical, legged systems due to the presence of a free scaling parameter $\alpha_F$. These findings were then verified using three separate reduced-order models of legged locomotion, demonstrating the preservation of dynamic similarity via the general scaling relationships, the requirement of the gravitational constraint on the general dynamic scaling laws for systems under the influence of gravity, as well as need to scale all dimensional parameters, regardless of whether they are physical parameters, control parameters or any other type.
In addition to verifying the utility of dynamic scaling in simulation, the application of dynamic scaling to the RHex and DynoClimber families was considered. In particular, it was observed that even though the dynamic scaling relationships derived herein were not available for the development of scaled versions of these systems, the system parameters closely match those predicted via the scaling laws. This is particularly striking when considering the RHex and EduBot platforms, as independent optimization routines were utilized for the two systems. These results support the hypothesis that dynamic scaling could prove an effective tool for streamlining the design of scaled platforms by eliminating the need for time and resource-intensive optimizations. In addition, several considerations related to the practical implications of dynamic scaling on actuator and material selection were addressed and the utility of dynamic scaling for preserving locomotive performance in gravitationally-altered environments was discussed.

While scaling is an important consideration in the design of dynamical, legged platforms, characterization of the performance of the resulting systems is equally significant. Quantification of stability for these platforms, while crucial for establishing performance specifications, has proved challenging and no widely accepted means of experimentally determining dynamic stability has been proposed. To address this concern, a variety of metrics relevant to the quantification of stability were reviewed, as well as potential indicators of performance behavior, particularly considering quantification techniques and difficulty (e.g. leg vs body, type of sensors, etc.). After paring the list of potentially viable metrics, simulation studies were undertaken to establish the efficacy of the various metrics and indicators in measuring the dynamic stability of SLIP-like systems. Using this study as a guide, a similar investigation was undertaken on an experimental hopping platform to ascertain the effectiveness of the metrics/indicators for a physical system. After considering the results of both studies, it was determined that the Gait Sensitivity Norm and Decay Ratio were most successful at measuring dynamic stability when utilized in conjunction with the proper gait indicators ($\beta^{LO}$ for GSN and $E^{TD}$ for DR).

While considering the tools for the development of multi-modal, legged systems, such as dynamic scaling and stability quantification, is crucial to producing effective platforms, the ultimate demonstration of success is the development of a platform capable of dynamical
locomotion via multiple modalities. To this end, dynamical, quadrupedal robot, SCARAB, was developed. A series of characterizations of the performance on both vertical and level surfaces were carried out, demonstrating a strong adherence of the platform to the underlying biologically-inspired templates, namely the LLS and FG models, as well as mean running speeds of $0.67ms^{-1}$ and mean climbing speeds of $0.16ms^{-1}$ were recorded. Furthermore, an initial set of modifications have been made, leading towards the capacity for autonomous transitions between running and climbing modalities using a minimal set of parameter alterations.

### 6.2 Future Endeavors

While many tools and principles relating to the development of multi-modal platforms have been addressed in this work, additional inquiries related to this topic are still warranted.

With regards to dynamic scaling, the newly derived scaling relations have yet to be used in a predictive manner to demonstrate their efficacy on experimental systems. In particular, the availability of the free $\alpha_F$ parameter is expected to provide a means for preserving dynamic similarity when changing system payloads by simply varying leg compliance. Another avenue for investigation relates to the degree of similarity required between platforms as well as parameter sensitivity to slight deviations in dynamic similarity.

As far as dynamic stability is concerned, while the set of metrics were examined with regards to a SLIP-like runner, the application to other legged modalities is still unconfirmed. In addition, though the GSN and DR were shown to correlate to MSV, the use of these metrics in a predictive study or in an optimization routine would further establish the utility of these metrics. Furthermore, while a variety of metrics were considered, this list was not all-inclusive and consideration of alternative metrics may yield even better quantification of dynamic stability.

Finally, though SCARAB has been shown to be effective at moving dynamically in both vertical and level regimes, it is still quite restricted when considering its readiness for field-testing. A key point of progress would be to integrate the capacity for autonomous behaviors and transitions, rather than require tethered operation and manual adjustment. This would
include the addition of an on-board power source, mechanisms for self-reconfiguration between running and climbing modalities and claws capable of active or passive adjustment to attach to various running and climbing substrates. Furthermore, though the vertical and level behaviors were characterized, neither quasi-static nor dynamic transitory behaviors were addressed, which will be necessary for field operation. Finally, on the current version of SCARAB, the control is quite limited with only a clock-based fixed-frequency controller utilized. Consideration of more complex controllers in addition to onboard intelligence for steering and path planning would improve the versatility of this platform.
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BIOGRAPHICAL SKETCH

Bruce D. Miller was born September 29, 1986 in Atlanta, GA. He enrolled at Boston University in September of 2005 for his undergraduate education and graduated magnacum laude with a Bachelor of Science in Biomedical Engineering in May of 2009. He was accepted into the B.S./Ph.D. program in Mechanical Engineering at Florida State University and enrolled in the Fall of 2009. Since beginning his studies at Florida State University, he has worked in the STRIDe Lab as a graduate student researcher, focusing on the study of dynamical legged robots, with particular consideration towards biologically-inspired design of such systems.